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Quantum deleting and signalling

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Abstract

It is known that if one could clone an arbitrary quantum state then one could send signals faster than the speed of light. Here, we show that deletion of an unknown quantum state for which two copies are available would also lead to superluminal signalling. However, the (Landauer) erasure of an unknown quantum state does not allow faster-than-light communication.

Keywords: Linearity; Cloning; Deleting; Signalling

A deep-rooted concept in quantum theory is the linear superposition principle which follows from the linearity of the equations of motion [1]. Linear superposition of states is the key feature which elevates a two-state system into a qubit. The possibility of exploiting greater information processing ability using qubits is now being investigated in the emerging field of quantum computation and information technology [2]. Further, linear evolution makes certain operations impossible on arbitrary superpositions of quantum states. For example, one of the simplest, yet most profound, principles of quantum theory is that we cannot clone an unknown quantum state exactly [3,4]. Indeed, stronger statements may be made with stronger assumptions: unitarity of quantum evolution requires that even a specific pair of non-orthogonal states cannot be perfectly copied [5]. If we give up the requirement of perfect copies then it is possible to copy an unknown state approximately by deterministic cloning machines [6–11]. Recent work shows that non-orthogonal states from a linearly independent set can be probabilistically copied exactly [12,13] and can evolve into a superposition of differing numbers of copy states [14].

Notwithstanding the above, we might ask: what could go wrong if one were to clone an arbitrary state? In 1982 Herbert argued that the copying of half of an entangled state, such as by a laser amplifier, would allow one to send signals faster than light [15]. That same year the no-cloning theorem demonstrated the flaw in this proposed violation of causality [3,4]. Thus, the linear evolution of even non-relativistic quantum theory and special relativity were not in contradiction. In fact, one can go a step further and ask if the no-signalling condition (the impossibility of instantaneous communication) lies behind some of the basic axiomatic structure of quantum mechanics [16]. It turns out that the achievable fidelity of imperfect

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cloning follows from this no-signalling condition [17, 18]. Further, it can be shown that even probabilistic exact cloning cannot violate the no-signaling condition [19].

Thus, quite a bit is understood about cloning machines, but what about other hypothetical machines? Recently, it was proved that given two copies of an unknown quantum state we cannot delete one copy against the other by any physical operation (i.e., by a trace preserving completely positive transformation)—a result called the 'no-deletion theorem' [20]. This is yet another fundamental consequence of the linearity of quantum theory and is not restricted to the class of unitary operations. The deletion of quantum information (in this sense) should not be confused with its erasure. Classically, erasure is a primitive operation that irreversibly resets a system into a standard state. This erasure involves a thermodynamic cost since the erased information appears as heat in the environment—a result known as Landauer erasure principle [21,22]. Quantum mechanically, erasure would allow for the resetting of even a single qubit. By comparison, deletion is the quantum analog of uncopying a bit of information from two identical bits. The essential difference is that irreversible erasure naturally carries over from the classical to the quantum world, whereas the analogous uncopying of classical information is impossible for quantum information. The quantum no-deleting principle has also been generalized to higher-dimensional quantum systems and for non-orthogonal states using unitarity in a conditional manner [23]. Even though one cannot delete one of a pair of non-orthogonal states perfectly using only unitary operations one can perform deletion of linearly independent states in a probabilistic manner [24,25]. Recently, a 'stronger no-cloning theorem' has shown that to copy a state from a non-orthogonal set, that the full information about the clone must already be provided in the ancilla state [26]. It has been suggested that these stronger no-cloning and no-deleting theorems taken together imply a property of 'permanence' to quantum information.

In this Letter we ask the question: suppose one can delete an arbitrary state using a quantum deleting machine, what could go wrong? We show that if one could delete unknown states then one could send signals faster than light! At first glance this may be surprising as by deleting information we are only reducing the redundancy at our disposal and that should not in any way affect the signal being sent. On the other hand, however, we know that the linearity of quantum theory has survived to its highest precession test and that even a little bit of non-linearity would allow superluminal signalling. Thus one can perhaps understand that since no-deleting is a consequence of linearity then any process that violates linearity should clash with one of the corner stones of special relativity. Therefore, deletion of an arbitrary state should lead to signalling. Finally, we show that the erasure of quantum information does not imply signaling, as expected.

First, let us recall the quantum no-deletion principle. Consider two copies of an unknown qubit $|\psi\rangle$ each in a Hilbert space $\mathcal{H} = \mathbb{C}^2$. The two copies live in a three-dimensional symmetric subspace of $\mathcal{H} \otimes \mathcal{H}$. The quantum no-deleting principle states that it is impossible to design a machine that can delete one copy even in the presence of a second identical copy of the unknown quantum state. That is, there is no linear transformation $\mathcal{L}: \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3 \to \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$ that will take

$$|\psi\rangle_1 |\psi\rangle_2 |A\rangle_3 \to |\psi\rangle_1 |\Sigma\rangle_2 |A_\psi\rangle_3, \tag{1}$$

where $|\Sigma\rangle$ is the blank state which can be of our choice, $|A\rangle$ is the initial and $|A_\psi\rangle$ is the final state of the ancilla. A properly working deleting machine must have the final state of the ancilla independent of $|\psi\rangle$ to exclude swapping, however, it will be convenient to include such a possible dependence for the moment. It has been shown by linearity alone that the only possible operation of the form of Eq. (1) is equivalent to swapping the unknown state onto the Hilbert space of ancilla [20,23]. However, this operation cannot be considered as proper deletion in the sense given above but is in fact (Landauer) erasure [20,22,23].

To show that deletion of an arbitrary state implies signalling consider the following scenario: let Alice and Bob be remotely separated and share two pairs of EPR singlets $|\Psi^-\rangle_{12}$ and $|\Psi^-\rangle_{34}$. (Note that in proving cloning implies signalling one uses only a single EPR pair shared between Alice and Bob.) Alice has particles 1 and 3 and Bob has 2 and 4. Since the singlet state is invariant under local unitary operation $U_i \otimes U_j$, $(i = 1, 3,\ j = 2, 4)$ it is same in all basis [up to U(1) phase factors]. Let us write the combined state of the system in an arbitrary (real) qubit basis

$\{|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle,\ |\bar{\psi}\rangle = \sin\theta|0\rangle - \cos\theta|1\rangle\}$ as

$$|\Psi^-\rangle_{12}|\Psi^-\rangle_{34} = \frac{1}{2}\big(|\psi\rangle_1|\psi\rangle_3|\bar{\psi}\rangle_2|\bar{\psi}\rangle_4 + |\bar{\psi}\rangle_1|\bar{\psi}\rangle_3|\psi\rangle_2|\psi\rangle_4 - |\bar{\psi}\rangle_1|\psi\rangle_3|\psi\rangle_2|\bar{\psi}\rangle_4 - |\psi\rangle_1|\bar{\psi}\rangle_3|\bar{\psi}\rangle_2|\psi\rangle_4\big). \tag{2}$$

First we notice that the state of the Bob's particles 2 and 4 are in a completely random mixture. Suppose now Alice were to measure her particles 1 and 3 into the qubit basis $\{|\psi\rangle, |\bar{\psi}\rangle\}$. If the outcome were $|\psi\rangle_1|\psi\rangle_3$, then after communicating the result to Bob, Bob's description for the state of particles 2 and 4 would be $|\bar{\psi}\rangle_2|\bar{\psi}\rangle_4$. If instead Alice's outcome were $|\bar{\psi}\rangle_1|\bar{\psi}\rangle_3$, then after receiving this information Bob's description for his particles would be the state $|\psi\rangle_2|\psi\rangle_4$. Similarly, one can find the resulting states with other choices of measurements. However, whatever measurements Alice does, if Bob does not learn the results his description will for them will remain that of a completely random mixture, i.e., $\rho_{24} = \frac{1}{4}I_2 \otimes I_2$. That is to say that local operations on Alice's subspace $\mathcal{H}_1 \otimes \mathcal{H}_3$ have no effect on the Bob's description of the state in the subspace $\mathcal{H}_2 \otimes \mathcal{H}_4$. Indeed, as is well known, the result of any measurement (von Neumann or POVM) that Bob can perform on his particles will depend only on the reduced density matrix of the particle 2 and 4.

But suppose Bob has a quantum deleting machine which can delete an arbitrary state. The action of the quantum deleting machine on the two copies and the ancilla state belonging to the Hilbert space $\mathcal{H}_2 \otimes \mathcal{H}_4 \otimes \mathcal{H}_5$ can be described by

$$\begin{aligned}
&|\psi\rangle_2|\psi\rangle_4|A\rangle_5 \to |\psi\rangle_2|\Sigma\rangle_4|A_\psi\rangle_5,\\
&|\bar{\psi}\rangle_2|\bar{\psi}\rangle_4|A\rangle_5 \to |\bar{\psi}\rangle_2|\Sigma\rangle_4|A_{\bar{\psi}}\rangle_5,\\
&|\psi\rangle_2|\bar{\psi}\rangle_4|A\rangle_5 \to |\phi'\rangle_{245},\\
&|\bar{\psi}\rangle_2|\psi\rangle_4|A\rangle_5 \to |\phi''\rangle_{245}.
\end{aligned} \tag{3}$$

The last two transformations correspond to the situation when the states are non-identical and in these cases the output can be some arbitrary states. After passing through the quantum deleting machine the combined state of Alice and Bob becomes

$$\begin{aligned}
&|\Psi^-\rangle_{12}|\Psi^-\rangle_{34}|A\rangle_5\\
&\quad\to \frac{1}{2}\big(|\psi\rangle_1|\psi\rangle_3|\bar{\psi}\rangle_2|\Sigma\rangle_4|A_{\bar{\psi}}\rangle_5\\
&\qquad + |\bar{\psi}\rangle_1|\bar{\psi}\rangle_3|\psi\rangle_2|\Sigma\rangle_4|A_\psi\rangle_5\\
&\qquad - |\bar{\psi}\rangle_1|\psi\rangle_3|\phi'\rangle_{245} - |\psi\rangle_1|\bar{\psi}\rangle_3|\phi''\rangle_{245}\big)\\
&\quad= \big|\Psi^{(\mathrm{out})}\big\rangle_{12345}.
\end{aligned} \tag{4}$$

Suppose Alice and Bob have pre-agreed that the measurements onto basis states $\{|0\rangle, |1\rangle\}$ means '0' and onto any other (say) $\{|\psi\rangle, |\bar{\psi}\rangle\}$ means '1'. Now, Alice performs measurements onto either of these two choices of basis states but does not communicate the measurement outcome. Since Bob is ignorant of Alice's measurement, he traces out the particles at Alice's lab and the ancilla at his lab too. The reduced density matrix for particles 2 and 4 is given by

$$\begin{aligned}
\rho_{24} &= \mathrm{tr}_{135}\big(\rho^{(\mathrm{out})}_{12345}\big)\\
&= \frac{1}{4}\big(I_2 \otimes |\Sigma\rangle_{44}\langle\Sigma| + \rho'_{24} + \rho''_{24}\big),
\end{aligned} \tag{5}$$

where

$$\rho^{(\mathrm{out})}_{12345} = \big|\Psi^{(\mathrm{out})}\big\rangle_{12345}\big\langle\Psi^{(\mathrm{out})}\big|,$$
$$\rho'_{24} = \mathrm{tr}_5(|\phi'\rangle_{245}\langle\phi'|)$$

and

$$\rho''_{24} = \mathrm{tr}_5(|\phi''\rangle_{245}\langle\phi''|).$$

Since $|\phi'\rangle_{245}$ and $|\phi''\rangle_{245}$ are in general pure entangled states of the non-identical inputs and the ancilla, they will depend on the input parameters. After tracing out the ancilla, we will have, in general, mixed entangled states given by

$$\begin{aligned}
\rho'_{24}(\theta) &= \frac{1}{4}\Big(I_2 \otimes I_4 + \vec{m}'(\theta).\vec{\sigma}_2 \otimes I_4\\
&\qquad + I_2 \otimes \vec{n}'(\theta).\vec{\sigma}_4 + \sum_{ij} C'_{ij}(\theta)\sigma_{i2} \otimes \sigma_{j4}\Big),\\
\rho''_{24}(\theta) &= \frac{1}{4}\Big(I_2 \otimes I_4 + \vec{m}''(\theta).\vec{\sigma}_2 \otimes I_4\\
&\qquad + I_2 \otimes \vec{n}''(\theta).\vec{\sigma}_4 + \sum_{ij} C''_{ij}(\theta)\sigma_{i2} \otimes \sigma_{j4}\Big).
\end{aligned} \tag{6}$$

Thus, it is clear that the reduced density matrix of particles 2 and 4 are no longer completely random and instead depend on the choice of basis. This shows that if Alice measures her particles in the $\{|0\rangle, |1\rangle\}$ basis then the density matrix of Bob's particles will be in $\rho_{24}(0)$. If Alice measures her particles in $\{|\psi\rangle, |\bar{\psi}\rangle\}$ basis then Bob's particles will be described by a different density matrix $\rho_{24}(\theta)$. Since these two statistical mixtures are non-identical Bob can distinguish them. Therefore, by *deleting an arbitrary state* he can distinguish the two statistical mixtures and that *will allow communication of one classical bit superluminally*. It is known that if one allows non-linear operations one can distinguish two statistical mixtures [27]. This suggests that the action of deletion was probably a non-linear operation beyond the realm of quantum theory.

Furthermore, we can show that erasure of unknown state does not imply superluminal signalling. Landauer erasure of information can be accomplished by swapping the last qubit with a standard state and then dumping it into the environment. Suppose Bob performs this erasure operation on the particles at his disposal. In this case, Bob can simply choose the initial state of the ancilla $|A\rangle$ to be the blank state $|\Sigma\rangle$. Then, he swaps the last two qubits in $\mathcal{H}_4 \otimes \mathcal{H}_5$ and traces over Hilbert space $\mathcal{H}_5$. Now instead of transformation (3) we have

$$\begin{aligned}
&|\psi\rangle_2|\psi\rangle_4|\Sigma\rangle_5 \to |\psi\rangle_2|\Sigma\rangle_4|\psi\rangle_5,\\
&|\bar{\psi}\rangle_2|\bar{\psi}\rangle_4|\Sigma\rangle_5 \to |\bar{\psi}\rangle_2|\Sigma\rangle_4|\bar{\psi}\rangle_5,\\
&|\psi\rangle_2|\bar{\psi}\rangle_4|\Sigma\rangle_5 \to |\psi\rangle_2|\Sigma\rangle_4|\bar{\psi}\rangle_5,\\
&|\bar{\psi}\rangle_2|\psi\rangle_4|\Sigma\rangle_5 \to |\bar{\psi}\rangle_2|\Sigma\rangle_4|\psi\rangle_5. \qquad (7)
\end{aligned}$$

Using the argument as before, without any communication from Alice to Bob, the two particle density matrix of Bob's particles (after swapping and tracing over the ancilla) is given by

$$\rho_{24} = \frac{1}{2} I_2 \otimes |\Sigma\rangle_{44}\langle\Sigma|. \qquad (8)$$

Basically Bob has transformed the state of particles 4 and 5 as $I_4/2 \otimes |\Sigma\rangle_5\langle\Sigma| \to |\Sigma\rangle_4\langle\Sigma| \otimes I_5/2$ and as a result he has dumped $\log 2$ bits of information to the environment in accordance with the Landauer erasure principle. This density matrix does not carry any information about Alice's choice of basis as it is independent of the parameter θ. Therefore, by erasing the information Bob will not be able to know onto which basis Alice has performed her measurement. Thus, as expected, the (Landauer) erasure of an unknown state does not lead to superluminal signalling.

The quantum no-deletion theorem is a consequence of the linearity of quantum theory. We have shown that violation of no-deletion can lead to superluminal signalling using non-local entangled states. However, (Landauer) erasure of information does not allow for any signalling. These two observations further illustrate the fact that quantum deletion is fundamentally a different operation than erasure.

We conclude with a remark that classical information is physical but has no permanence. By contrast, *quantum information is physical and has permanence* (in view of the recent stronger no-cloning and no-deleting theorems in quantum information [26]). Here, permanence refers to the fact that to 'duplicate' quantum information the copy must have already existed somewhere in the universe and to 'eliminate' it, it must be moved to somewhere else in the universe where it will still exist. It would be interesting to see if the violation of this permanence property of quantum information can itself lead to superluminal signalling. That it should be true is seen here partly (since deleting implies signalling). It remains to be seen whether negating the stronger no-cloning theorem leads to signalling.

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