

Perfect Simulation for Tame Markov Chains

STEPHEN CONNOR (joint work with WILFRID KENDALL)

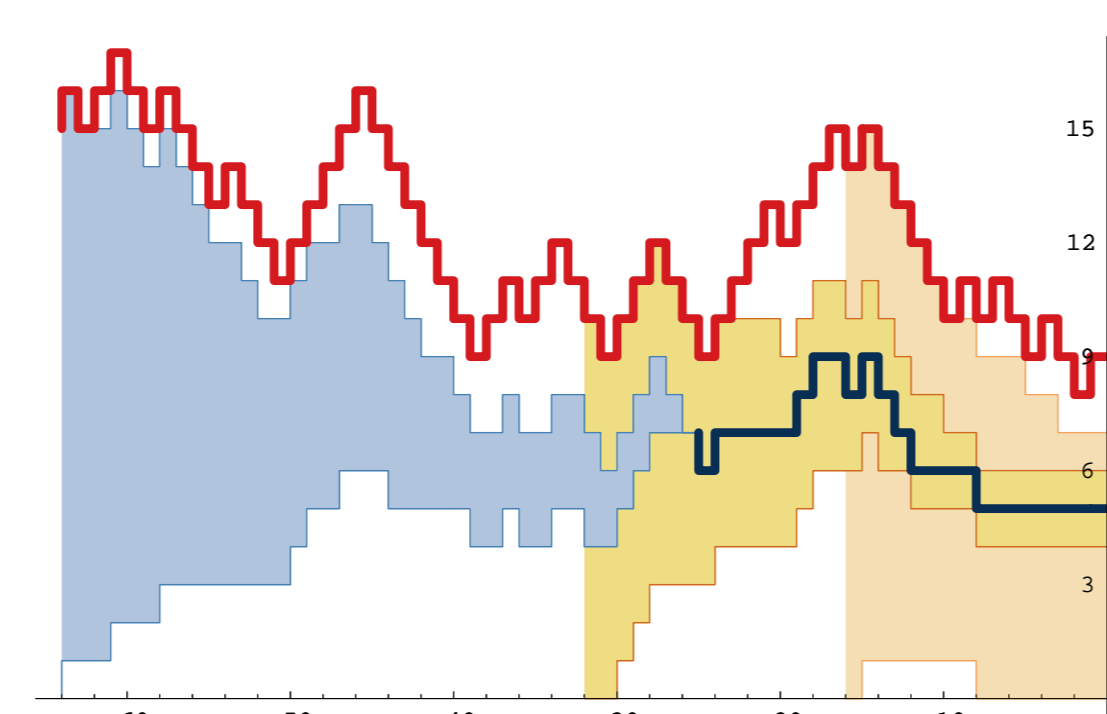
Department of Statistics, University of Warwick, UK

1. Perfect simulation for rapidly converging chains

Perfect simulation algorithms sample from the *exact* stationary distribution of a Markov chain X . Unfortunately, the best-known of these algorithms, *Coupling from the Past* (CFTP) [1], can (in theory) be used if and only if X is *uniformly ergodic* [2].

However, a *Dominated CFTP* (domCFTP) algorithm exists for any chain X which is *geometrically ergodic* [3]. Under mild regularity, X then satisfies a *drift condition* of the form

$$\mathbb{E}[X_1 | X_0 = x] \leq \beta x + b\mathbf{1}_{[x \leq d]}, \quad \beta \in (0, 1). \quad (\star)$$



domCFTP in practice

Condition (\star) and sub-sampling permit construction of a *dominating process* Y for X ($\log Y$ is simply the workload of a $D/M/1$ queue) – and hence a domCFTP algorithm.

2. Subgeometrically ergodic chains

Subgeometrically ergodic chains satisfy a weaker drift condition, such as

$$\mathbb{E}[X_1 | X_0 = x] \leq x - cx^\alpha + b\mathbf{1}_{[x \leq d]}, \quad \alpha \in (0, 1). \quad (\star\star)$$

This drift condition does not produce a positive-recurrent dominating process. But consider a new class of chains [4]:

Definition. The chain X is called *tame* if, given $X_0 = x$, there exists a *taming function* $F(x) \propto [x^\delta]$ with $\delta \in (0, 1)$, such that

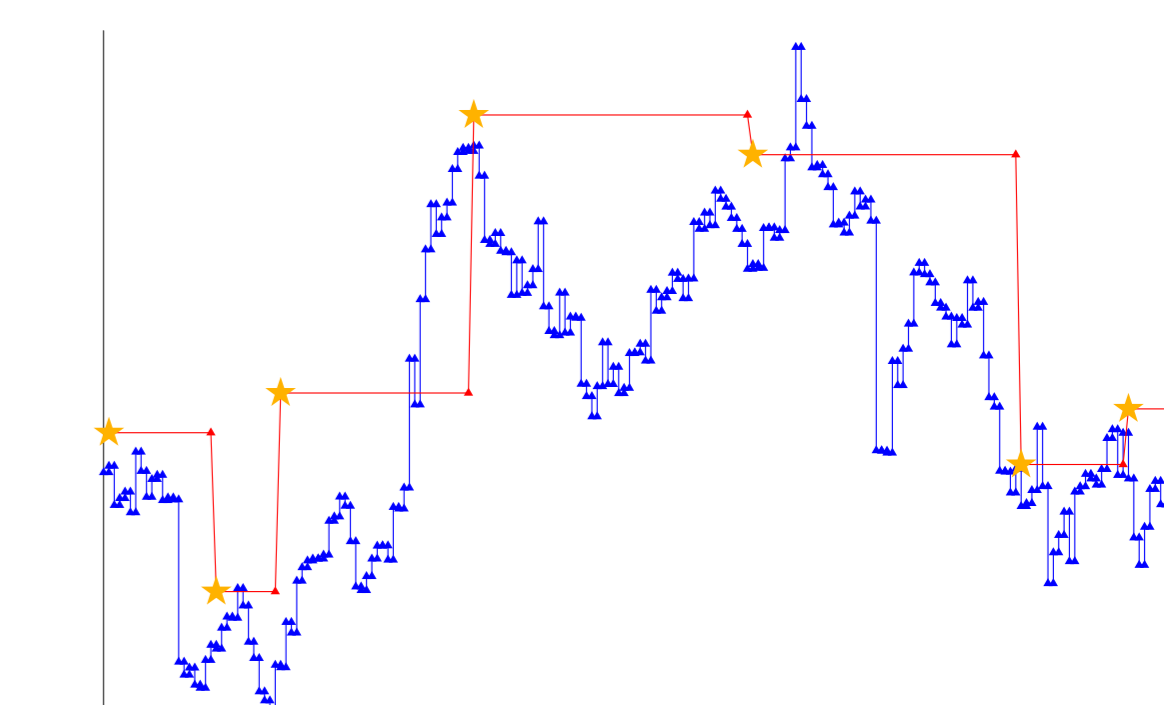
- $\mathbb{E}[X_{F(x)} | X_0 = x] \leq \beta x + b'\mathbf{1}_{[x \leq d']}$
- $\log \beta < \delta^{-1} \log(1 - \delta)$ ($\star\star\star$)

Use F to *adaptively sub-sample* X to produce a geometrically ergodic chain X' ($X'_0 = x, X'_1 = X_{F(X'_0)}, \dots$ etc).

3. domCFTP for tame chains

Since X' is geometrically ergodic, it has a dominating process Y . Use the function F to *pause* Y , yielding a process D *dominating* X *when* D *moves*.

- The bounds in $(\star\star\star)$ yield a proper equilibrium distribution for D ;
- D is easy to simulate in equilibrium and in reversed-time;
- This is enough to produce a domCFTP algorithm for X [4].



Delayed dominating process for X

(This algorithm is not practical in general, but directly extends the results for uniformly and geometrically ergodic chains: in practice a dominating process better suited to X would be used.)

4. Tame chains: do they exist?

YES! Many chains have been shown to be tame [4], including:

- any chain X satisfying drift condition $(\star\star)$ with $\alpha > 0.7$;
- a number of birth-death processes and random walks on \mathbb{R}^+ ;
- any *Random-walk Metropolis-Hastings algorithm* on \mathbb{R}^d with a suitably heavy-tailed target density and symmetric, compactly supported proposal density.

QUESTIONS:

- Can the bound $\alpha > 0.7$ be improved?
- Does there exist a polynomially ergodic chain that is not tame (a *wild chain*)? The hunt continues...



References

- [1] Propp, J.G. and D.B. Wilson (1996). *Random Structures Algorithms* 9.
- [2] Foss, S.G. and R.L. Tweedie (1998). *Stochastic Models* 14.
- [3] Kendall, W.S. (2004). *Electron. Comm. Probab.* 9.
- [4] Connor, S.B. and W.S. Kendall (2007). *Ann. Appl. Probab.* 17(3).