To appear in Physics World, March 2006

The Root of the Matter: Voting in the EU Council

by

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The distribution of votes in the Council of Ministers of the European Union was a major obstacle during the negotiations of the Treaty Establishing the Constitution for Europe, and remains one of the factors hindering the current process of ratification of the Constitution. In 1952 the British psychiatrist and mathematician Lionel Sharples Penrose (father of the eminent British scientists: mathematician Sir Roger, and physicist Oliver) made a prophetic statement predicting the flaws that are now inherent in the EU Constitution:

> "[…] if two votings were required for every decision, one on a per capita basis and the other upon the basis of a single vote for each country, the system would be inaccurate in that it would tend to favour large countries […]"

The mathematical theory of voting put forward by Lionel Penrose in the mid 1940's, combined with proposals arising from research carried out by the authors at the Jagiellonian University in Kraków in 2004, bring about an unexpectedly simple and effective solution to the problem of voting in the EU Council.

The enlargement of the EU to 25 (and in the not too distant future 27) countries makes it necessary to adopt new voting rules in the Council of Ministers. The provisions differ considerably between the Treaty of Nice, currently in force, and the Treaty Establishing a Constitution for Europe, signed in Rome in October 2004, pending ratification.

Pursuant to the Treaty of Nice, the voting weights of individual member states in the Council of Ministers reflect to a certain degree the population of each state. The Council adopts a piece of legislation if:

- the sum of the weights of the member states voting in favour exceeds 232 (approximately 72% of the total weight of all 25 member states);
- the member states constituting a qualified majority represent at least 62% of the total population of the Union;
- a majority of member states (at least 13 out of 25) vote in favour of the motion.

According to the Treaty Establishing a Constitution for Europe, a resolution would be adopted by the Council of Ministers subject the following criteria:

- at least 55% of member states, comprising at least 15 of them, vote in favour;
- the member states voting in favour comprise at least 65% of the total population of the Union;

- additionally, a blocking minority must include at least four Council members, failing which a qualified majority shall be deemed attained.

The main deficiency of the Nice system is that it proves extremely difficult to form a winning coalition and to pass any new legislation efficiently. The Nice system is complicated by the fact that three diverse criteria need to be applied simultaneously in the calculation of qualified majority. In addition, as we shall see, it assigns too much power to certain countries, while others obtain less power than appropriate. Meanwhile, the European Constitution assigns too much power to the largest and the smallest states, while middle-sized countries do not receive their due share of influence.

Is it possible to construct a two-tier voting system free of these deficiencies, in which each citizen of each member state would have the same voting power? Can it be done in a way that is transparent, easy to implement, efficient to use, and will readily accommodate any future extensions of the Union?

A partial answer to this question can be found in Penrose's work, who deliberated in 1946 on the principles of an ideal representative voting system (in the context of a hypothetical distribution of votes in the UN General Assembly). First consider direct elections of the Government of a member state with population size N. The average citizen of a large member state, such as Germany, has less influence on the election of the Government of his or her country than, for example, a citizen of the much smaller Luxembourg. Penrose established that in such elections the voting power of a single citizen is proportional to $1/\sqrt{N}$, given that votes are uncorrelated. The elected Government then nominates the Minister to vote on behalf of the country in the European Council. It follows that the system of indirect voting applied to the European Council would be representative, giving each citizen of each EU state the same influence on the decisions of the Council, if the voting power of each country were proportional to $\sqrt{N}$, cancelling out the factor $1/\sqrt{N}$.

This principle of distribution of voting power in a wider context of a two-tier voting system is known as the *square root law of Penrose*. There exists a close analogy in physics: the mean square distance travelled by a diffusing particle is proportional to the square root of time. From the mathematical viewpoint the square root laws concerned with voting power and diffusion follow very similar reasoning.

Note that Penrose's law can, if desired, be stated without explicit reference to the square root. It is sufficient to stipulate that the voting weight of each member state should be equal to the side of a square with area equal to its population.

A voting system in the EU Council of Ministers that obeys Penrose's square root law was first proposed in 1998 by Annick Laruelle from the Université Catholique de Louvain, Belgium and Mika Widgrén from the Turku School of Economics and Business Administration, Finland. Since then such voting systems have been endorsed by scientists from the Czech Republic, France, Germany, Israel, Norway, Poland, Spain, Sweden, Switzerland, and the United Kingdom.

The Penrose law provides a theoretical solution to the question of how to allocate voting power to member states. However, until recently it had remained unclear in general how to attribute voting weights and how to define qualified majority decision rules in order to obtain the desired distribution of voting power.

In this connection, it is important to make a clear distinction between the voting weight of a given member state and their voting power. The two quantities do not have to be proportional. In a hypothetical Union of two states, with voting weights 51% and 49% operating under a simple majority decision rule, the country with the higher voting weight would be able win every vote, taking 100% of the voting power. Between 1958 and 1973, despite a relatively high voting weight assigned to Luxemburg as compared to its population, it was in fact impossible for this country exercise any a priori voting power in the European Economic Community Council. The configuration of the voting weights of the remaining member states was such that Luxemburg was unable to influence the outcome of any ballot, irrespective of the vote cast by their representative.

A country can only exercise power whenever their vote cast in a ballot proves decisive: should this country decide to change its vote, the winning coalition would fail to satisfy the qualified majority condition. Voting power can be understood as the ratio of the number of coalitions in which the vote of a given country is decisive to the number of all possible coalitions in which it takes part. This is known as the Banzhaf index, after John F. Banzhaf III, the well-known American lawyer and practitioner of public interest law, who introduced this ratio as a measure of voting power independently of Penrose in 1965.

In the EU consisting of 25 states there are more than 33.5 million of possible coalitions. When two more states join the Union in the near future, this number will increase to over 134 million. Given the voting weight of each state and the qualified majority rules, it is then possible to compute the number of coalitions in which the vote of a given state proves decisive, and to work out the voting power of that state as measured by the Banzhaf index.

In early 2004 two of the current authors proposed a voting system based on the following criterion:

- the voting weight of each member state should be allocated proportionally to the square root of its population, with decisions passed by the Council if the sum of weights exceeds a certain quota, namely 62% for the EU of 25 countries and 61.4% for 27 countries.

This particular choice of the quota would ensure that the voting power of each country is practically equal to the voting weight and, as a result, proportional to the square root of the population. The Penrose law would then be fulfilled almost exactly, and the potential influence of every citizen of each member state on the decisions taken by the Council would be very nearly the same, irrespective of the size of the state. Such a voting system would not only be representative, but also transparent: the voting powers of member states would become proportional to the voting weights, and only a single criterion would be needed for the calculation of qualified majority. Moreover, any further enlargement of the Union would involve a one-off recalculation of a single quantity, the quota used in the qualified majority rule.

This proposed voting system stimulated considerable interest among experts in voting theory. It has since been christened by the media as the “Jagiellonian Compromise,” with overtones referring to the Jagiellonian University in Kraków, where the research was carried out, and to the Jagiellonian dynasty, who reigned over vast parts of Central Europe during the Renaissance period. Prior to the June 2004 EU Summit in Brussels, an open letter in support of square root voting weights in the EU Council endorsed by eminent European scientists,

predominantly mathematicians and physicists, was sent to the Governments of EU member countries and EU institutions.

Figure 1 illustrates the effects of the provisions of the European Constitution, the Jagiellonian Compromise and the Treaty of Nice on the voting power of individual citizens of six representative EU member states with widely differing population sizes: Germany (82.5 million citizens), the United Kingdom (59.3 million), Poland (38.2 million), the Netherlands (16.2 million), the Czech Republic (10.2 million) and Lithuania (3.5 million). It shows that voting power is very unequally distributed among the citizens of different countries under the Treaty of Nice as well as under the European Constitution. As predicted by Penrose, the citizens of Germany, with its large population, would gain considerable additional voting power if the Constitution were to be adopted.

What is the reaction by politicians? The Jagiellonian Compromise has been endorsed by number of leading politicians in Poland as part of their agenda. The former Irish Prime Minister John Burton has made numerous positive references to the voting system based on the square root law. A recent article by David Reid in *Physics World* referring to the Jagiellonian Compromise was quoted and the voting system scrutinised in the House of Commons Library Research Paper No 04/75. When a similar system was put forward by Swedish diplomacy in 2000, the Swedish Prime Minister Göran Persson commented: 'Our formula has the advantage of being easy to understand by public opinion and practical to use in an enlarged Europe…it is transparent, logical and loyal. Maybe that is why it does not please everybody.'

If, as is now increasingly likely, the Constitution for Europe will fail to come into force, the question of the voting system in the EU Council of Ministers will need to be revisited. If a solution is sought based on rational principles to ensure equal distribution of voting power among all citizens of European Union countries, then the principles underlying Penrose's square root law and the Jagiellonian Compromise may be able to inform future discussions and negotiations.

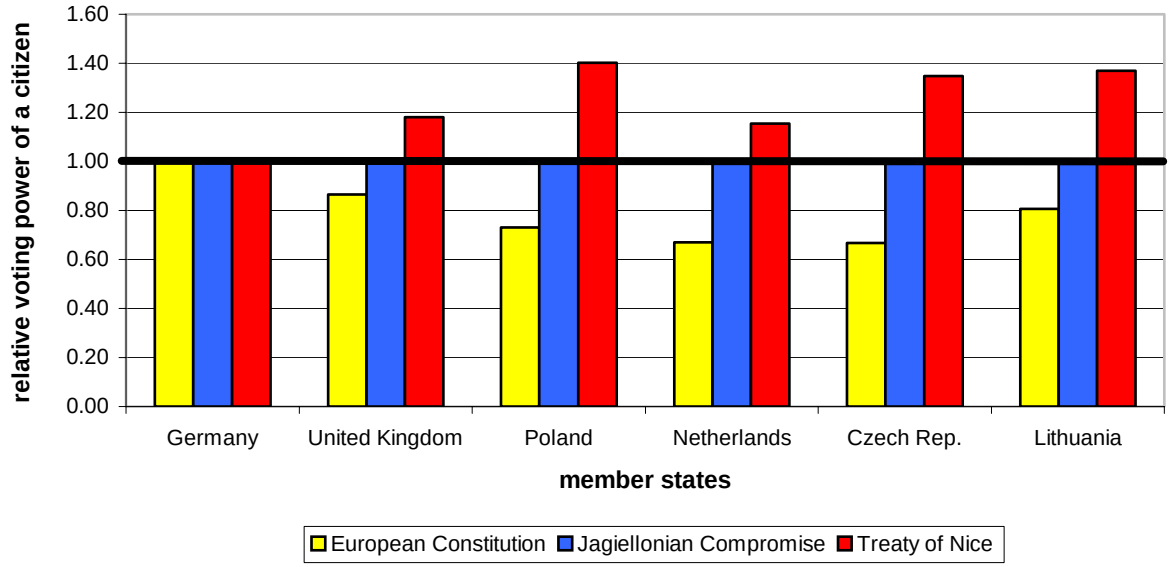

Fig. 1. Voting power of a citizen of the United Kingdom, Poland, the Netherlands, the Czech Republic and Lithuania under the European Constitution, the Jagiellonian Compromise and the Treaty of Nice relative to the voting power of a citizen of Germany.

Further reading

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