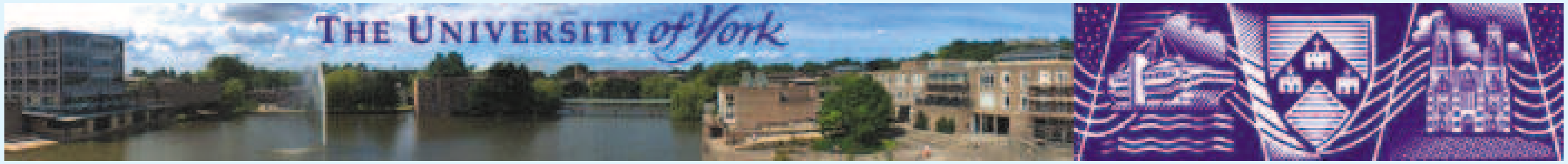

Dynamical Systems and Applications

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1. Motivation Behind the DSA Module

This is the study of the mathematical modelling of systems that evolve with time.

The main emphasis in the module is the continuous time systems. In real life applications, it is easier to record the rate of change of observables and this leads to a differential equation.

Explicit solutions are rare to find and there is a need to consider a qualitative study of these models (instead of quantitative).

2. DSA Module Syllabus

1. One-dimensional continuous time systems and bifurcations

2. Planar linear systems

3. Planar non-linear systems:

- Fixed points, stability, linearisation.
- Conservative systems and “energy”.
- Index theory.
- Limit cycles.

4. Bifurcations of planar systems

- Saddle node, transcritical, pitchfork and Hopf bifurcations.

5. Lorenz systems

3. Masters Mini-Project

Extra lecture will be given by ZC on bifurcations (with supervision given over the Autumn term) to lead to the students to write a mini-project.

Main Reference: S.H. Strogatz, *Nonlinear Dynamics and Chaos*, Westview Press, 1994, York Library Code S7.38 STR.

4. Examples

Competing Species.

1. Consider the system:

$$\begin{cases} \dot{x} &= x\,(1 - x - y) \\ \dot{y} &= y\,(0.5 - 0.25y - 0.75x) \end{cases} , \qquad (1)$$

when x and y are non-negative.

There are four equilibrium points, namely, $(0, 0)$, $(1, 0)$, $(0, 2)$ and $(0.5, 0.5)$.

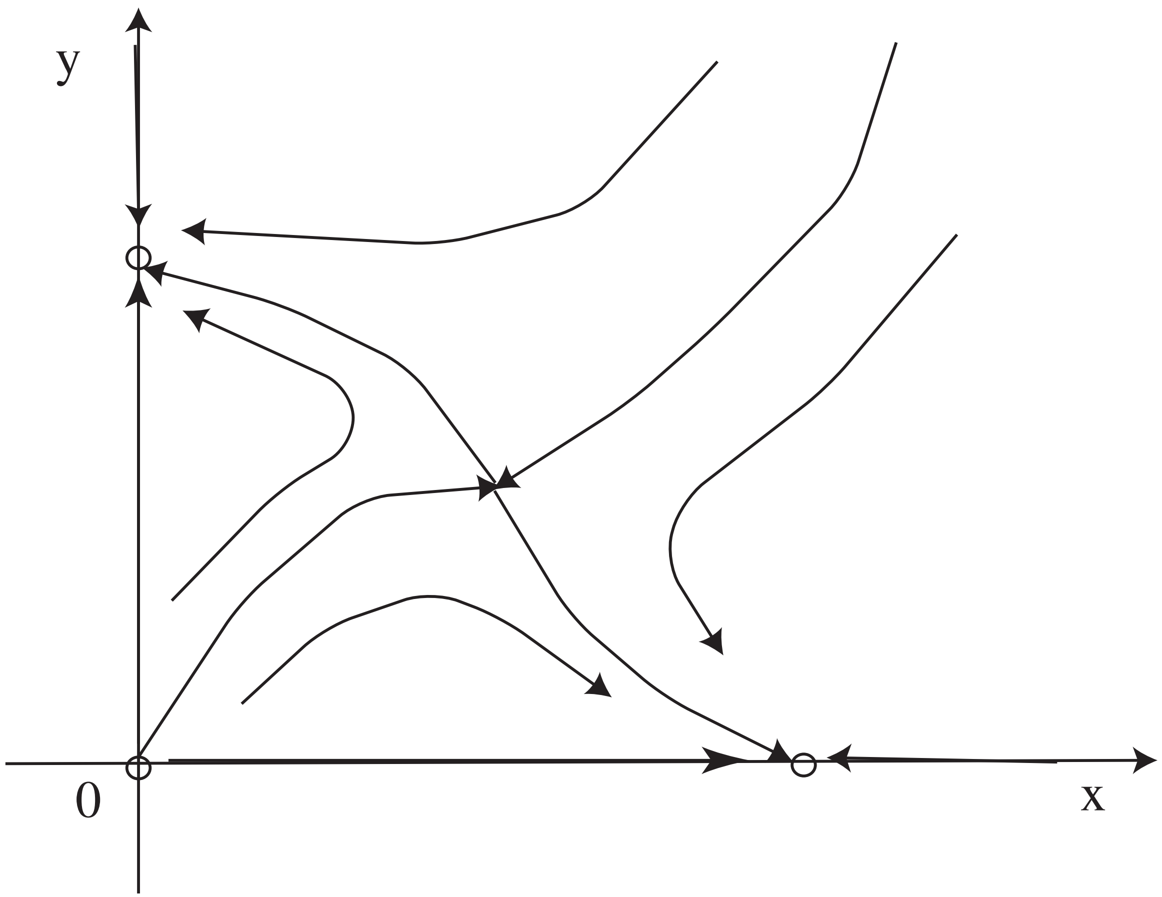


Figure 1: Phase portrait of the system (1).

2. Consider the system:

$$\begin{cases} \dot{x} = x(1 - x - y) \\ \dot{y} = y(0.75 - y - 0.5x) \end{cases}, \tag{2}$$

when x and y are non-negative.

There are four equilibrium points, namely, $(0, 0)$, $(0, 0.75)$, $(1, 0)$ and $(0.5, 0.5)$.

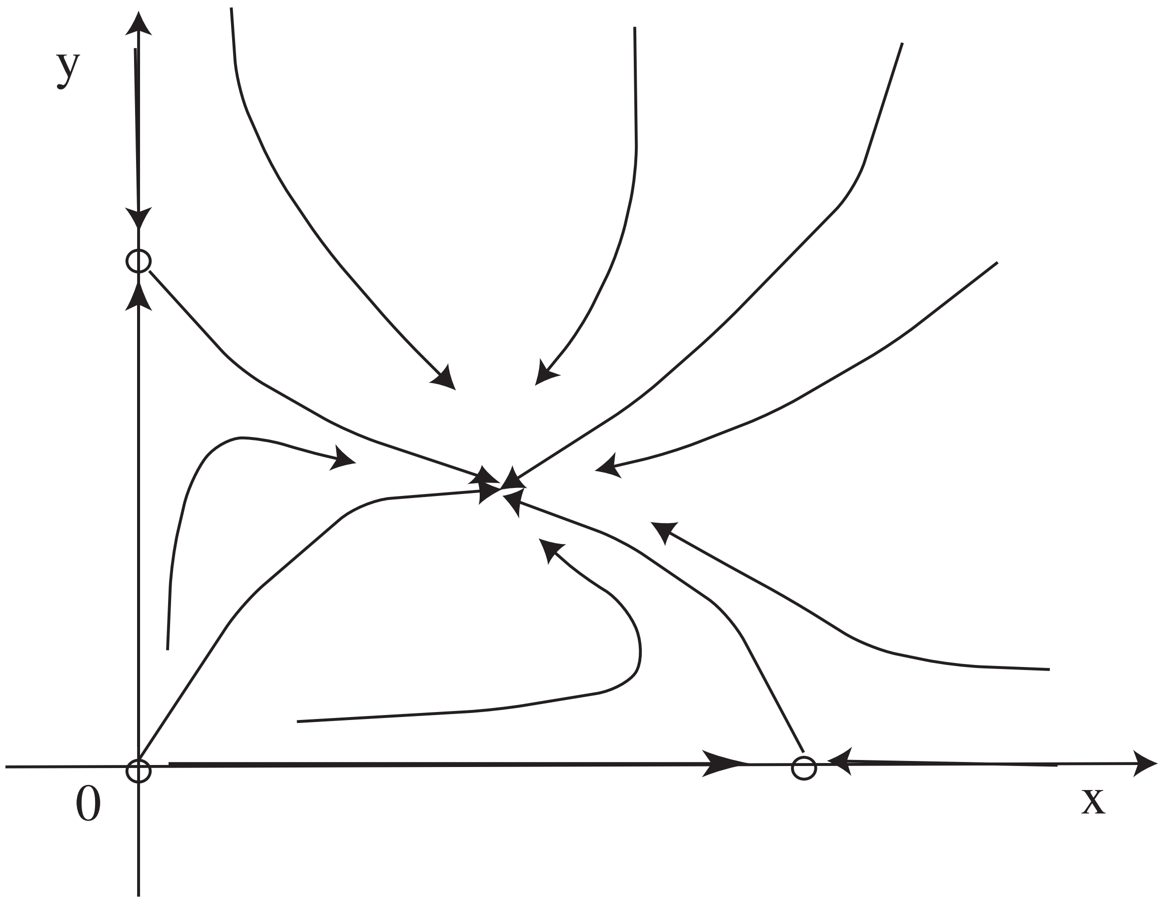


Figure 2: Phase portrait of the system (2).

3. Consider the system:

$$\begin{cases} \dot{x} = x\,(1 - 0.5y) \\ \dot{y} = y\,(-0.75 + 0.25x)\ , \end{cases} \tag{3}$$

when x and y are non-negative.

There are only two equilibrium points, namely, $(0, 0)$ and $(3, 2)$.

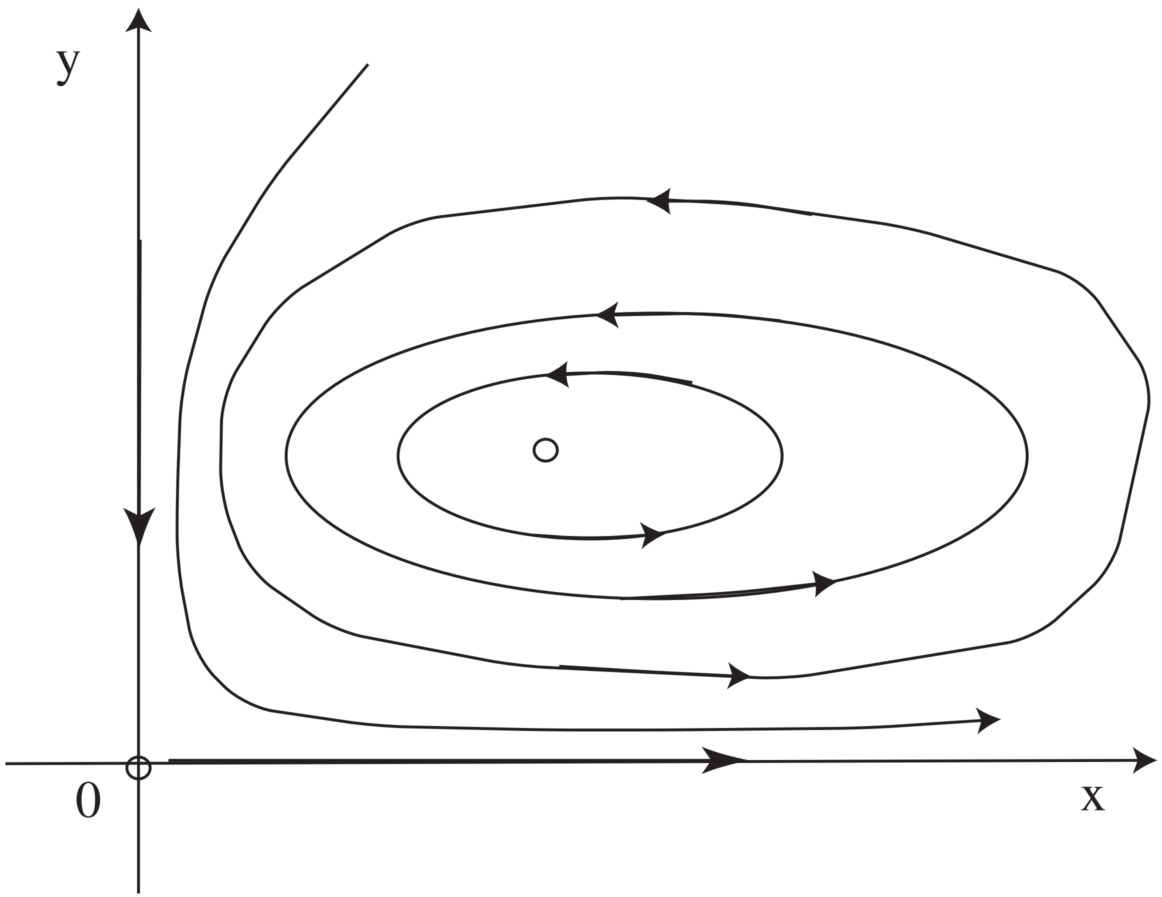


Figure 3: Phase portrait of the system (3).

5. Lorenz Systems

$$\begin{cases} \dot{x} &= \sigma(y - x) \\ \dot{y} &= rx - y - xz \\ \dot{z} &= xy - bz \end{cases}$$

Lorenz observed chaotic behaviour for $\sigma = 10$, $b = 8/3$ and $r = 28$. A experimental picture is shown in the next slide.

The equilibrium points are the solutions to the corresponding vector field being zero, i.e. setting the equations:

$$\begin{cases} \sigma(y - x) = 0 \\ rx - y - xz = 0 \\ xy - bz = 0 \ . \end{cases}$$

The origin is always a solution. If $0 < r \leq 1$ then the origin is the only solution. However, if $r > 1$ then there are two more equilibrium points, namely

$$C_{\pm} = \big(\pm \sqrt{b(r-1)}, \pm\sqrt{b(r-1)}, r - 1 \big) \ .$$

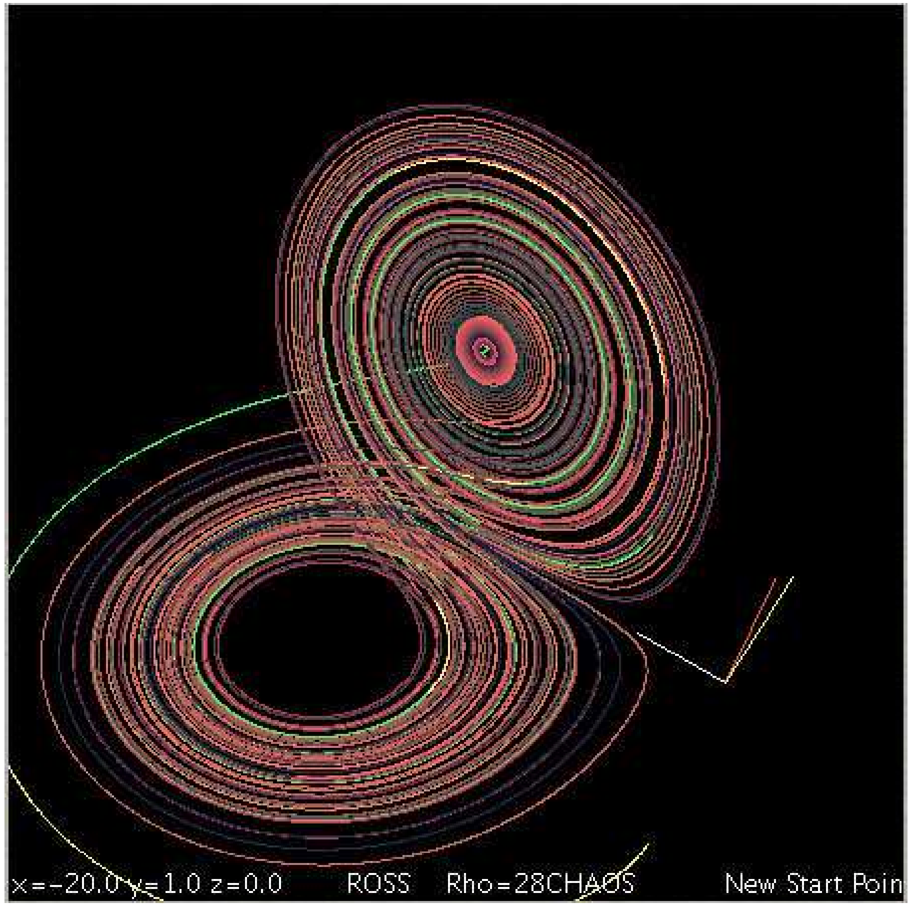
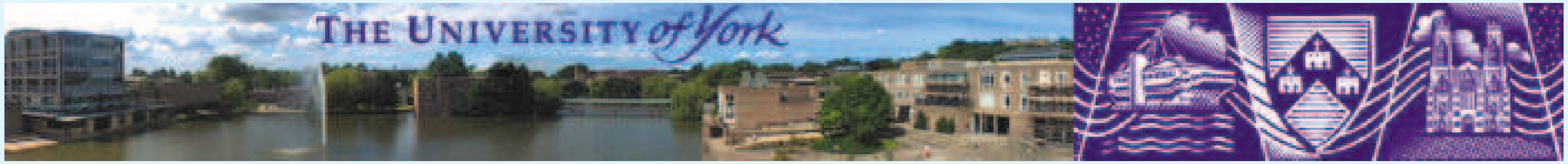


Figure 4: An experimental example of the Lorenz attractor. Taken from the webpage: `http://to-campos.planetaclix.pt/fractal/lorenz_eng.html`



Thank you.

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