# Pair production at moderate optical depths

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**Abstract**. Electron-positron pair production must be included when considering the hard X-ray spectra of AGNs and $\gamma$-ray bursters. We investigate thermal plasmas with temperatures $kT \simeq m_e c^2$ and optical depths $1 \lesssim \tau \lesssim 5$, including pair processes, Comptonization and bremsstrahlung. Analytic and numerical results are presented for equilibrium and impulsively heated plasmas.

## 1. Introduction

The presence of electron-positron pairs can have important consequences for AGNs, $\gamma$-ray bursters and other high luminosity, compact sources. Pairs increase the cooling rate and the opacity of the source, and affect radiation transport.

Much work, analytical and numerical, has been done on timescales, radiation processes and pair equilibria in constant temperature plasmas [5]–[8] [10]–[14]. Here we discuss plasmas of moderate optical depth, $\tau \gtrsim 1$, where Comptonization is an important source of hard photons. We present an analytical discussion of both constant temperature and impulsively heated plasmas, and we describe the results of a computer program developed to model inhomogeneous plasmas in a fully time-dependent manner.

We quote energies and temperatures in terms of the electron rest mass, a natural unit for the problem under consideration. A photon of energy $\omega$ = 1 has a physical energy of 511 keV, and a particle temperature of $T$ = 1 corresponds to $T_*$ = 5.9 × $10^9$ K. We parameterize the gas by the "proton optical depth", $\tau_p = N_p \sigma_T R$, where $N_p$ is the proton number density, $\sigma_T$ is the Thomson cross section, and $R$ is the physical size of the system. Our unit of time is the "proton Thomson time", $t_p = 1/N_p \sigma_T c = R/c\tau_p$. In physical units $t_p$ = $5000 \times (N_p / 10^{16} m^{-3})$ seconds. $z$ is the ratio of pairs to protons.

## 2. Constant Temperature Plasmas

For a plasma with given optical depth and temperature, what is the equilibrium pair density? The limit of small optical depth, where Comptonization can be neglected, has been discussed elsewhere [10]. Here we discuss the other limit: high optical depth and a photon spectrum significantly modified by Comptonization. We make the approximation that all the bremsstrahlung photons with $\omega > \omega_{\min}$ (see later) are Comptonized up to an energy $\omega \sim T$ before they escape. In equilibrium, pair production balances annihilation. We make the further approximation that photon escape balances the production of hard photons via Comptonization of the internally produced soft bremsstrahlung photons; we neglect the annihilation photons. This is valid provided that the annihilation photons do not get down-scattered below the pair production threshold before they produce a pair, i.e. provided the temperature is high enough, $kT \gtrsim m_e c^2 / 2$. (In the detailed numerical calculation the annihilation photons are included properly.)

There are two processes that determine $\omega_{\min}$, the energy below which bremsstrahlung photons can be ignored: photons with energies less than $\omega_{\min}$ either cannot scatter up to $\omega \sim T$ before escaping, or suffer bremsstrahlung self-absorption.

We approximate the bremsstrahlung spectrum of a single electron by

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$$\frac{dn^{ff}(\omega)}{dt} \simeq \begin{cases} 0 & \omega < \omega_{\min} \\ f(T) & \omega_{\min} < \omega < T \\ 0 & T < \omega \end{cases}$$
$$f(T) = \begin{cases} 10\alpha_f\left(\sqrt{T} + 1/\sqrt{T}\right) & T < 1 \\ 20\alpha_f & 1 < T \end{cases} \tag{1}$$

The number of *Cb* photons produced per proton per Thomson time is then

$$\frac{dn^{Cb}}{dt} \sim (1+2z)^2 f(T)\ln(T/\omega_{\min}) \equiv (1+2z)^2 N^{Cb} \tag{2}$$

$N^{Cb} \simeq 2$ for $T = 1, \omega_{\min} = 10^{-7}$.

Balancing the production of hard *Cb* photons with photon escape:

$$\frac{dn}{dt} = (1+2z)^2 N^{Cb} - \frac{n}{(1+2z)\tau_p^2} = 0 \tag{3}$$

The pair production and annihilation rates (neglecting annihilation photons) are balanced in equilibrium:

$$\frac{dz}{dt} = \left(n\,e^{-1/T}\right)^2 - \frac{z(1+z)}{1+T^2} = 0 \tag{4}$$

The factor of $\exp(-1/T)$ gives the exponential cutoff in the photon-photon rate at low temperatures (below threshold). The factor of $1/(1+T^2)$ gives the correct low and high temperature limits of the annihilation rate.

Eliminating $n$ gives an expression for $dz/dt$ in terms of $z$, $T$ and $\tau_p$. $dz/dt = 0$ has either two or no solutions; the high optical depth analogues of the two solutions found by Svensson [10]. The criterion for an equilibrium solution to exist is

$$\tau_p \lesssim \frac{0.4}{\sqrt{N^{Cb}}}\frac{e^{1/2T}}{(1+T^2)^{1/4}} \tag{5}$$

The actual values of $\tau_{crit}(T)$ should not be taken too seriously, since we have been neglecting factors of order 2 throughout. The functional dependence, however, is real. To get more accurate values of $\tau_{crit}$ we use a numerical method. The model includes Comptonization, photon transport, thermal bremsstrahlung, annihilation and pair production; it is described in more detail elsewhere [1] [2] [4] [9].

The region with stable solutions is shown in figure 1. The output spectrum of a typical equilibrium model is harder than any observed $\gamma$-ray spectrum, as far as we are aware, and has no annihilation feature.

# 3. Impulsively Heated Plasmas

The constant temperature case is simple to analyse since it depends on two parameters, $T_e$ and $\tau_p$, and energy balance need not be considered. Next simplest is an impulsively heated gas which is then allowed to cool freely. We assume that the protons are "instantly" heated (i.e. on a timescale very much shorter than any other relevant timescale) to a temperature of a few tens of $m_e c^2$. They then heat the electrons (we assume by Coulomb interaction) which Compton cool off their own bremsstrahlung. Pairs are produced in the process. How efficiently is the large initial thermal energy of the protons converted into pairs?

This problem also depends on two parameters (to within the accuracy of our assumptions), this time $T_p$ and $\tau_p$. But now energy balance must be included. The initial electron temperature, $T_e^{init}$, is determined by saying that the protons rapidly heat the electrons until Compton cooling balances Coulomb heating. An initial proton temperature of a few tens of $m_e c^2$ can support a maximum electron temperature of about an $m_e c^2$.

To get the time dependence, and to find the maximum pair density, $z_{max}$, we have to solve simultaneously the four equations for pair production, photon production, and electron and proton energy balance:

$$\frac{dz}{dt} = \left(n\,\mathrm{e}^{-1/T_e}\right)^2 - \frac{z(1+z)}{1+T_e^2} \tag{6}$$

$$\frac{dn}{dt} = N^{Cb}(1+2z)^2 - \frac{n}{(1+2z)\tau_p^2} - 2\frac{dz}{dt} \tag{7}$$

$$\frac{dE_p}{dt} = -10^{-2}(1+2z)(T_p - T_e)(1+T_e^{1/2})\big/T_e^{3/2} \tag{8}$$

$$\frac{dE_e}{dt} = -\frac{dE_p}{dt} - T_e N^{Cb}(1+2z)^2 \tag{9}$$

Annihilation photons are included in the photon production rate, since we are not in equilibrium.

Solving these equations (numerically) gives results shown in figure 2. The solutions are insensitive to $T_e^{init}$; the electron temperature rapidly rises to ~ $T_e^{max}$, before many pairs are produced. The dependence on $T_p$ is strong, but indirect; a lower $T_p$ results in a lower $T_e^{max}$, and so a lower pair production rate.

Since factors of ~ 2 have been neglected in deriving these results, again only their qualitative form should be taken seriously. However, we do expect a maximum pair density of a few, after a few tens of Thomson times. For more accurate results, we turn to our computer model. Including energy balance, we allow the temperature of the electrons and protons to vary with time and position in the gas.

The central temperatures and pair densities as a function of time are shown in figure 3. These vary in an identical manner to the solutions of equations 6–9. The spectra at early times, before much pair production has taken place, are simple Comptonized bremsstrahlung spectra. The instantaneous spectra at late times (figure 4) have have three regions: a low energy Comptonized bremsstrahlung spectrum $N(\omega) \propto \omega^{-1}$, a break to $N(\omega) \propto$ constant, due to the down scattering of annihilation and the original hard bremsstrahlung photons, and an exponential turnover at a few $kT_e$.

These thermal models seem unlikely to apply in astrophysical situations since the spectrum is much too hard. This is an inevitable feature of thermal models; bremsstrahlung has a very hard spectrum, which is made even harder by Comptonization.

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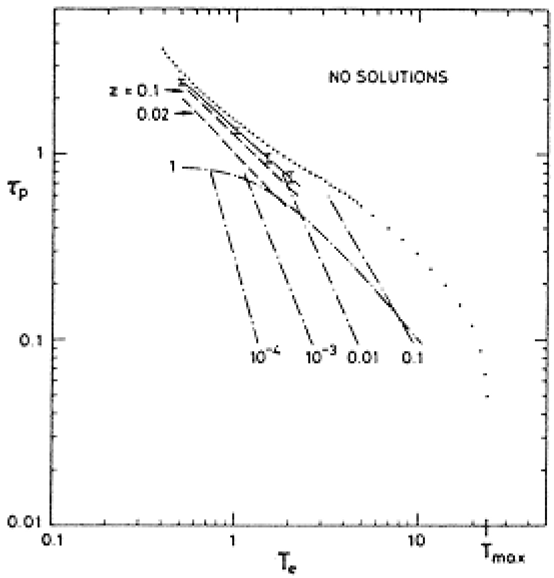


**Figure 1**. The $T_e$, $\tau_p$ plane. The solid line is the numerical result; the dotted line is from eqn 5, joined smoothly to the optically thin result; the dash-dotted lines are adapted from [10].

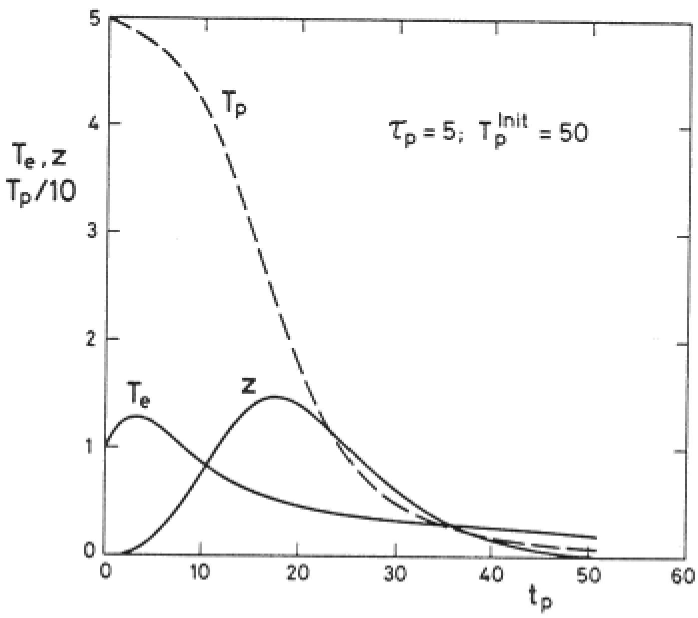


**Figure 2**. Solutions of eqns 6–9 with $T_p^{init} = 50$, $\tau_p = 5$, $T_e^{init} = 1$, taking $N^{Cb} = 0.5$, $E_p = 3\, T_p / 2$ and $E_e = 2\, T_e$.

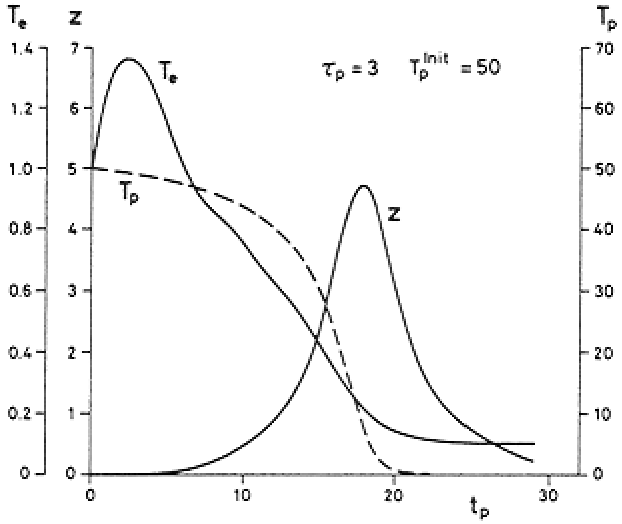


**Figure 3**. Temperatures and pair densities in a time dependent numerical model with $T_p$ = 50, $\tau_p$ = 3, $T_e$ = 1.

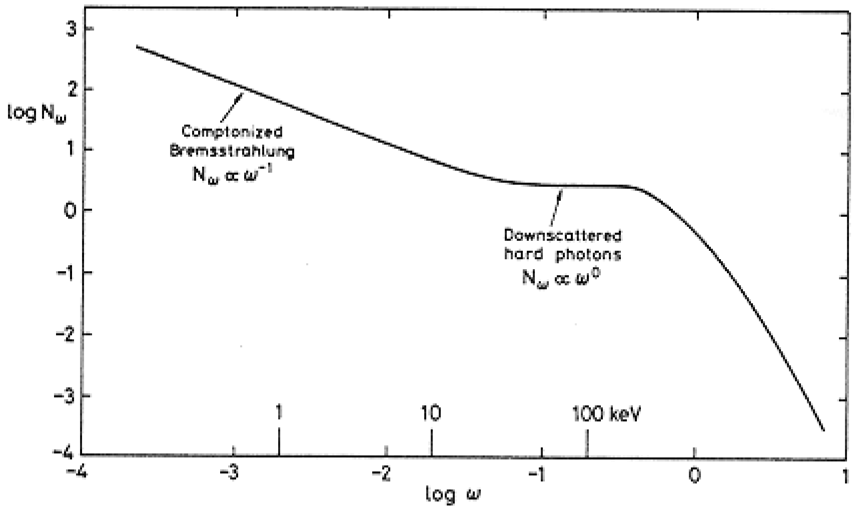


**Figure 4**. The output spectrum, photons per proton per Thomson time, from the model in figure 3.