Energy as a driver of diversity in open-ended evolution

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Abstract

We investigate the consequences of introducing an energy model into open ended evolutionary simulations. We propose a metamodel for simulations that incorporate an energy model and apply that model by extending Turk's *Sticky Feet* model. We show that introducing an energy model produces simulations with measurably increased diversity of the simulated population.

Introduction

We are interested in open ended evolution and in particular evolution within systems that are open to a simulated energy flux, open to changes in the simulated environment, and open to the representation of evolutionary mechanisms. In this paper we focus on energy flux, which allows us to represent many aspects of real world systems, such as the availability of food supplies, and different means of making a living within an environment, be they predatory or sessile.

In order to investigate these issues we have chosen to extend Turk's Sticky Feet [10] model. This gives a simple mechanism for implementing mobility and experimenting with open-ended evolution. A Sticky Feet simulation is a collection of simulated creatures moving in a 2D domain. Each such creature is a graph of *springs* connecting together *feet*. Motion is achieved as a consequence of simple harmonic oscillation of the springs, which pushes the feet around within the simulation space. The coefficient of friction experienced by the feet is modulated–at times slippy, at times sticky–which results overall in motion through the space.

Each creature has a heart and a mouth, each of which is a distinguished type of foot. The heart represents the creature's 'essence'. The mouth–when it happens upon another creatures's heart–allows the former creature to eat the latter, removing it from the simulation. The likelihood of a creature happening upon another is facilitated to some extent by the springs being equipped with sensors, which may modulate the oscillation of the spring when in the presence of another creature's heart. This allows a creature to turn towards another, with the chance that it might then be able to

consume the target. When a creature is consumed the eater produces a single offspring, which may be a mutation of the parent. Mutations that include additional feet, springs and sensors allow the creatures to evolve in a manner that eventually produces offspring that are better adapted to hunting for and eating other creatures.

A Sticky Feet world is one in which creatures evolve to improve their performance at consuming other creatures, and therefore being able to pass on their genome. As such, it provides some aspect of a model of open ended evolution. We use this term here in the sense of an evolutionary system where components continue to evolve new forms continuously, rather than halting when some 'optimal' or stable position is reached [9].

Sticky Feet [10] works in this manner, as there is no overall fitness function and all creature behaviour is expressed in a single large environment rather than relying on artificial two-creature tournaments. As such it is representative of many aspects of real-world evolution.

There is, though, no mechanism for sticky feet creatures to pass on their genomes other than by consuming other creatures. That is, the simulation is closed to the development of non-predatory behaviour. This is useful from the point of view of maintaining a constant sized simulation, but is not representative of real world evolution where population sizes can change dramatically.

Natural evolution–that which operates in the world around us–is different in essence from the sticky feet model in that success does not entirely derive from hunting and reproduction. Creatures in natural environments must be able to extract some sort of living from that environment, supported either by consuming other creatures, or by turning some flux in the world, for example sunlight or the chemical nutrients consumed by extremophiles, into food.

This argument is essentially that famously made by Malthus in 1798 [7], which led Darwin towards the principle of natural selection [2]. Although Malthus discussed the availability of food we generalise this to the availability of *energy*. This is a limited resource although the environment is continually bathed in an energy flux. This flux may
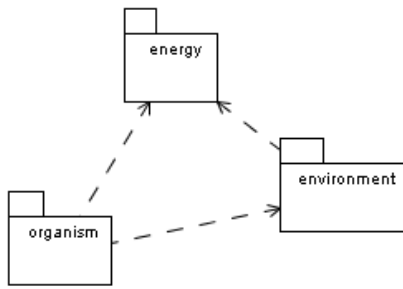
Figure 1: Domain metamodel

Figure 2: Energy metamodel

Figure 3: Environment metamodel

be used, and stored, by components of the environment, but if it is ignored it disappears and is no longer of use.

Natural systems are open: they are in receipt of some sort of resource flux such as that we model as energy. In this paper, we provide a meta-model for open simulations with energy flux, consumption, and storage; we describe an extended sticky feet simulation incorporating an instantiated energy model; we show that diversity is maximised when the flux is neither too low, nor too high.

Energy metamodel

In our work we use the CoSMoS approach [1]. We model the aspects of the domain that we wish to simulate as the *domain model*. We describe the actual simulation using the *platform model*, which executes on the *simulation platform*, producing results that can be analysed with respect to the *results model*.

In this paper we describe a *class* of models, ones that permit a particular sort of open ended evolution of sticky feet like creatures in a world, a domain, which is bathed in an energy flux. That is, we must define a *domain metamodel* to which our domain models must conform.

The domain meta model describes all possible domain models that we wish to explore, without limiting the particular domain. An abstract view of our meta-model is shown as figure 1[1] and shows the inter-dependencies of the three top level packages in our model: *Organism*, *Energy* and *Environment*.

Energy

Energy is modelled, as in figure 2, as a scalar quantity in arbitrary units. We also describe the *entropy* of some energy which might be thought of as the temperature of the energy which allows us to describe essential aspects of the energy economy. For example, in the natural world a continuous low flux of low entropy energy is available in the form of sunlight. Plants sequester this energy in a form that allows other organisms, such as animals to consume them and acquire the stored energy. Those animals subsequently excrete waste products which still represent energy, albeit in

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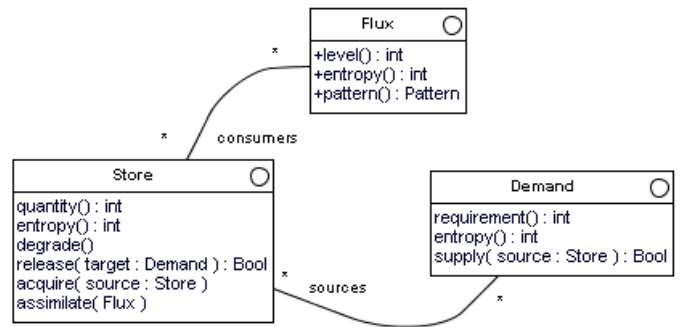[1]All the models here are expressed using the UML.
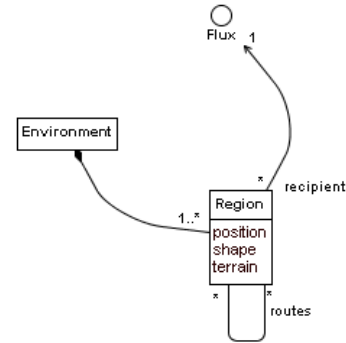
a higher entropy form, but which may still be metabolized by organisms such as dung beetles.

Although some authors use a simple "battery" model or conservation of energy (for example [3]), here we propose an energy model integrated with reproduction and behaviour.

Flux. The most basic part of the energy model, representing a flow of energy from outside the modelled system. This flux represents energy with a defined entropy and with a particular temporal pattern; for example at a high level during daytime but a much lower level during nighttime.

Store. One action of all members of a simulated world is to store energy. An organism might maintain its existence by consuming other stores, in the manner of herbivores eating plants, or by assimilating the flux itself as the plant itself does.

Demand. Many components of a simulated world make energy demands. Such components could be the physical structure of an organism, which requires energy to build and maintain, or an activity that an organism undertakes, such as hunting for other organisms to consume.

Environment

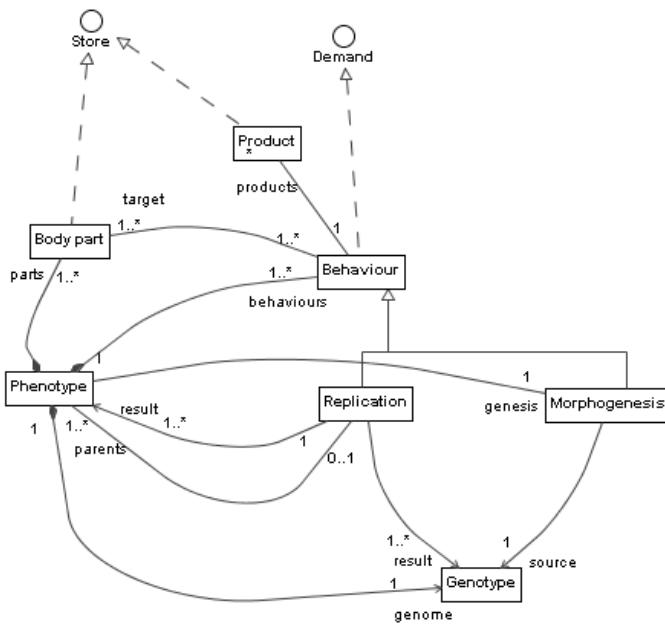The environment metamodel is elaborated in figure 3.

Figure 4: Organism Metamodel.

Environments are represented as a collection of Regions, each of which is the recipient of a particular Flux. Regions are connected together by routes each of which allows organisms to move from one region to another, albeit at a certain energy cost.

Organism

The organism metamodel is elaborated in figure 4[2]. It has two interdependent components: the phenotype and the genotype.

Genotype. The Genotype metamodel requires that an organism model expresses a genotype, which can be used as the source information for a morphogenesis process that grows its associated Phenotype. The genome of a phenotype is the result of a replication process that also creates a new Phenotype.

Phenotype. The Phenotype metamodel expresses that an organism's phenotype, its structure, consists of a number of *body parts* and a number of *behaviours*.

Body parts store energy: they realise the Store component of the energy model. The body parts are also the *target* of the organism's behaviours. For example, a bird's wings might be the target of its 'flying' behaviour. Each behaviour affects at least one part of an organism's body, and all such parts must be such a target of at least one behaviour.

Behaviours consume energy: they realise the Demand component of the energy model. We require that all energy

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[2]The arrowheads in this diagram refer to the UML property of *navigability* not to a notion of one object "producing" another.

consumption is expressed as a behaviour. So, for example, a purely sessile organism must still include a behaviour that it continually expresses, which demands the energy needed to maintain its metabolism. The energy for a behaviour is supplied by the body parts that are the target of the behaviour.

Some of an organism's behaviours produce waste products, included as the *Product* component. Such waste products are in themselves further energy stores, although they are not part of the organism's phenotype. The entropy of such waste products would usually be higher than that of the original energy source, but that does not preclude some organisms being able to scrape out an existence using such low grade sources of energy. A further waste product is the phenotype of a dead creature. Again this represents a low-grade source of energy, providing carrion-eating as a possible way of making a living in a world that conforms to our model.

All organisms possess the *Morphogenesis* behaviour; the genome contains the information needed for this behaviour. The specific genome of an organism is the result of another behaviour, *Replication*, which creates the genome of an offspring organism, potentially generating a mutated genome.

Discussion

Our metamodel expresses the essential requirements for evolution in an energetic context. A range of different implementations of this model are feasible. That is, a number of models could be produced, each of which conformed to this metamodel in the sense that the model's components were instances or realisations of components in the metamodel. Each such model would describe the domain model for a particular set of simulations in a particular domain.

Note that some ALife simulations incorporate a very basic notion of a constrained resource. Tierra [8] uses CPU time-slices as an analogue of energy, with the size of the time slice being a tunable function of the entity's size. However, there is no analogue of an energy store that would enable entities to 'time-shift' their use of the resource, or hand on a surplus to their progeny; Tierra is a 'use it or lose it' model. (Ray [8] mentions a possible extension allowing capture of CPU slices.) Stringmol [5] is an AChem with an explicit, but very simple, energy model: a fixed number of energy units are added to the container at each timestep, and molecules need to use an amount to execute each instruction. However, the energy is a global resource (energy is not stored in individual entities, but in the system and accessible to all). Our rich energy metamodel provides a number of features that organisms should be able to exploit to enable a range of different ways of making a living.

Energetic sticky feet

We have developed one simulation model (figure 5) that conforms to our energy metamodel. This is an 'energetic' variant of Turk's Sticky Feet [10]. That is, our energetic sticky feet model discusses the same sort of concepts that Turk
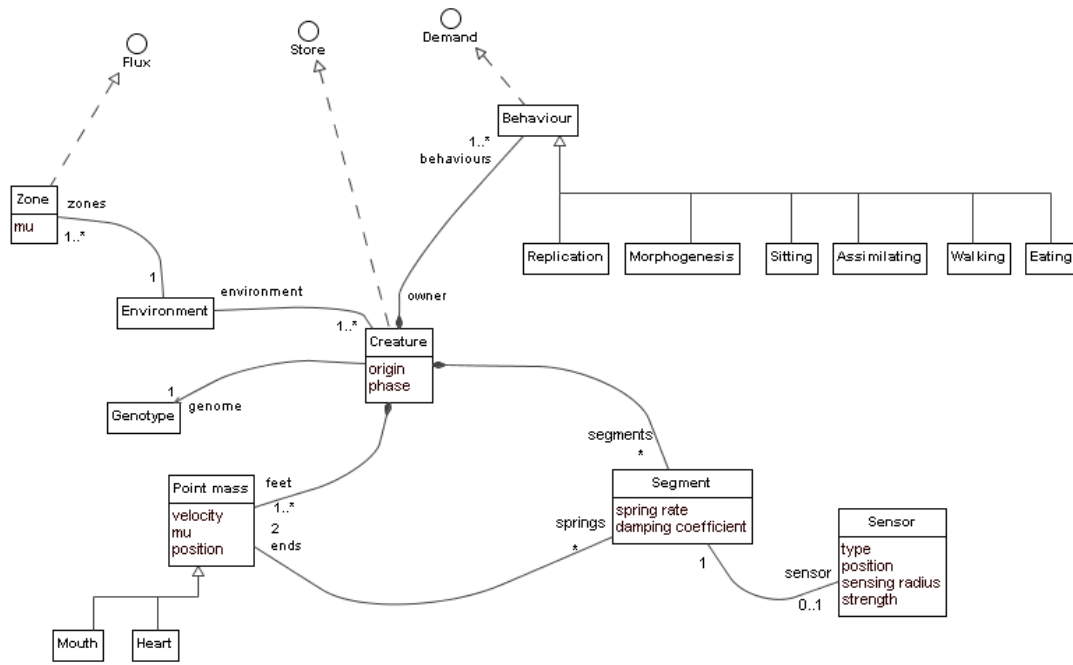
Figure 5: Energetic sticky feet overview

uses, albeit in the context of energy, environment and organism as prescribed by our metamodel.

Our experimental hypothesis is that the presence of the energy model will influence the evolution of the simulated creatures in such a manner that a more diverse world will result. We test this hypothesis by running the energetic sticky feet simulation for a range of flux levels, and compare the diversity of the evolved creatures with an unconstrained variant that ignores the need for energy.

Body parts

The creatures in our model follow the metamodel: each creature has a number of parts and a number of behaviours. The specific body parts are *feet* and *segments*. Following Turk [10]: a foot is a point mass with a particular, and modulatable, coefficient of friction; a segment is a spring that follows the equations from [10], to achieve motion due to simple harmonic oscillation of the springs as the point masses' coefficients of friction are varied.

The feet themselves appear in three varieties. The basic ones are augmented with special variants, representing a heart, and a mouth. The heart represents the 'essence' of a creature. When one creature's mouth gets close to the heart of another creature then the former may 'eat' the latter (assuming that the former creature is expressing the 'eating' behaviour).

Each segment may optionally have an attached sensor, which senses the position of other creatures. A sensor may sense either the heart or the mouth of another creature and, when it does, may perturb the oscillation of its attached seg-

ment. In this manner the sensors allow a creature to turn towards prey, or away from a predator.

Behaviours

The overall behaviour of each creature is represented by attaching a collection of individual behaviours to the creature. Each of these acts in a manner reminiscent of the *Command* pattern [4], and applies itself if it determines that the time is appropriate. Every behaviour demands energy, which must be provided by the owner of the behaviour. If the owner cannot supply the energy then the creature dies: it has exhausted its energy supplies.

Our energetic sticky feet implementation does not implement the waste product component of the metamodel. Consequently, when a creature dies, it just disappears from the simulation, taking with it any residual energy.

The behaviours available to a sticky creature are:

Sitting: the 'null' behaviour that all creatures must express. This behaviour forces a creature to continually consume energy. The amount of energy consumed is a function of the complexity of the creature's phenotype; a larger, more complex, creature requires more energy just to sit in one place compared to a small, simple, creature.

Walking: the behaviour that expresses the mode of walking explored by Turk [10], by oscillation of the creature's segments. The size of the energy requirement is proportional to the friction against which work is done by the springs.

Eating: the behaviour that allows a creature to look to see if any other creature's heart is in the vicinity of one of its mouths. If so, the former creature may 'eat' the latter. This adds to the eating creature's energy stores all of the energy of the eaten creature. The eaten creature is removed from the simulation.

Reproducing: the behaviour that allows a creature to create offspring, with a genome that is a mutation of the single parent's genome. At each simulation step there is a probability, encoded in the genome, that a creature may express this behaviour. We allocate energy costs to all the components of the phenotype, and check that the parent has sufficient energy to construct the child organism. If so, and the child organism is deemed to be viable, then it is created and the energy store of the parent is shared equally between the parent and the child.

Morphogenesis: the behaviour that is followed to construct the phenotype of a new organism from the genome generated by, optionally, mutating the genome of the organism's parent. This differs from the Reproducing behaviour in that it is reponsible for building the phenotype of the organism from its genome whereas the reproducing behaviour creates the new organism's genome.

Assimilating: the behaviour that allows an organism to gather energy directly from the flux in the current environment. The amount of energy available is determined by the flux applied to the region of the environment that the creature is inhabiting, and by the physical size of the creature. A larger creature, in the same manner as a large tree, can extract more energy from the flux, but needs correspondingly more energy to construct and maintain the larger phenotype.

Mutation and morphogenesis

In order to get some sort of evolution of the sticky feet creatures our implementation allows for mutation of the genome whenever the reproducing behaviour is expressed. Mutation is implemented by structuring a genotype as a sequence of genes, each of which codes for a particular part of the creature and its behaviour. Unlike Turk [10] we do not express a 'species' in any way in our model. Rather, each organism just has its own genome; even though it is likely that many other creatures have the exact same genome we do not use this in any part of our simulation. Following Turk's lead we implement two general forms of mutation, both of which are used in any individual mutation step. The first of these is the modification of the various parameters that apply to each component. For example this allows the position of the creature's feet, the stiffness of the springs in the segments, and the probability that a creature will attempt to express the reproducing behaviour at any particular point in time to be varied. The second form represents structural modifications of the phenotype. Specifically, these modifications may be
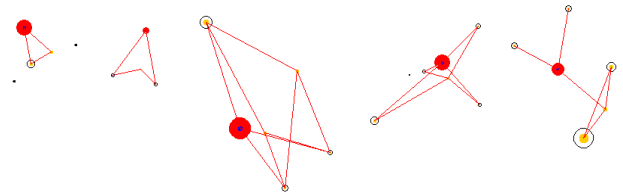

Figure 6: Some example evolved creatures; the filled circle is the heart, the open circles are mouths. From left to right these are: a) the initial 'seed' creature; b) the 'manta ray', only a few mutations away from the seed; it has two mouths; c) the 'killer', large and fast; d) the 'multimouth', with lots of mouths that stab outwards; e) the 'spiky', with lots of mouths but little area.

performed: adding feet or segments, removing feet or segments, adding a sensor to a segment and modifying a segment so as to connect to a different foot.

A possible result of one or more of these mutations is that the eventual creature does not form a viable phenotype. For example, it is possible to generate a genome that implies a phenotype where the feet and segments are not connected as a single structure, or where a creature does not have a heart. We choose to declare these mutations non-viable, and terminate the particular cycle of reproduction when they occur.

Even if a mutation represents a viable creature, it is possible that the resulting creature cannot be incorporated into the current simulation world. Specifically, in a similar manner to Turk, we do not allow phenotypes that initially overlap existing creatures. That is, our simulations are expressly two dimensional at the moment.

Viable creatures are created at a point in the simulation space that is local to their parent.

Implementation

Our energetic sticky feet implementation follows closely the model shown in figure 5. The implementation is written in pure Java and uses our environment-orientation approach [6] to represent the interaction of many creatures in a multi-threaded implementation. The environment is a two dimensional world with cyclic boundary conditions.

In all our experiments we initialise a simulation run with a fixed number of simple 'seed' creatures with a pre-defined genome and a random (according to a Gaussian probability distribution function) amount of energy. A typical collection of evolved creatures is shown in figure 6.

There are a large number of parameters to our sticky feet simulations. For example, there are parameters describing the construction energy required for each part of an organism, for the rate of mutation and for the level of energy flux in different regions of the environment.

Initial experiments with our implementation show that careful setting of these parameters is necessary in order to

allow the creatures to survive. That is, it is very easy to set the parameters so that there is insufficient energy in the environment for a population of creatures to survive; even though they can mutate to take advantage of their environment they run out of time in which to do so. This is in some ways perhaps a consequence of our approach of seeding the environment with a collection of fully formed creatures with significant energy demands.

Experimentation

In order to compare our simulations with something more representative of Turk's implementation [10] we need a way of 'turning off' the energy model. That is, we need to be able to run simulations in a manner that is not constrained by the availability of energy. In Turk's implementation the simulation has a fixed size population as a consequence of each creature reproducing once only when it consumes another. Hence, the simulation world does not get overrun with a vast number of creatures.

In a similar manner, our simulation includes an 'unconstrained energy' option where the creatures function exactly as they do in the energetic world except that the *demand* of all behaviours is set to zero, so no energy is ever consumed, and the *reproducing* behaviour is only available, and indeed is forced, in the situation where the *eating* behaviour has been invoked. This has the effect of creating a fixed-population simulation (except that on occasion a new creature cannot be 'fitted in' to the existing simulation, in which case reproduction is delayed until space is available) of a form similar to Turk's.

The differences between the implementation of the 'energetic' and 'unconstrained' variants of our simulation are minor. Hence, we can be sure that measured differences in the results of the simulations are a consequence of the inclusion, or exclusion, of the energy model.

In order to track the develop of creatures as they evolve we use a notion of *mutation distance* in our experiments. As discussed we have no specific notion of 'species' in our implementation. Rather, each creature has its own genome, which has a mutation distance. The initial population of creatures all have a copy of the same genome, which has *mutation distance* = 0. Whenever a creature reproduces it may also mutate the genome which is passed on to the child creature. The likelihood of allowing such a mutation is one of the simulation's parameters. After this mutation, following the process described earlier, the implementation compares the resulting genome with the initial genome. If they are different (they might not be because of the random nature of choosing whether to adopt specific mutations) and the genome represents a viable creature, then that new genome mutation distance is incremented.

In this manner every creature has a mutation distance, and we use this as part of our experimental results. There is not a simple relationship with *time*; it is possible, although un-

likely, for example, for a creature with mutation distance 150 to co-exist in a simulation with another of mutation distance 0. The latter creature could have survived from the outset–our creatures do not die of old age–or it could be the end result of a series of reproductions that involved no mutations.

Each creature in our simulations has an *area* that determines the amount of energy it receives from the environment's energy flux. We calculate its area by regarding the creature as an irregular polygon, ignoring feet that have only one attached segment, and by calculating the area of that polygon. So, a creature that was two feet connected by a single segment (a frequently occurring shape) would have *area* = 0 and would not receive any energy from the environmental flux.

Hypotheses

The direction of our experimentation is towards investigating two hypotheses. First, we hypothesise that creatures evolving in the context of an energy model should do so in a manner that is measurably different from that which applies in a 'unconstrained energy' world. Second, we hypothesise that the presence of the energy model creates a wider range of ways of the sticky feet creatures 'making a living'. For example, a creature could survive by eating other other creatures, or it could survive by growing large enough to acquire sufficient energy from the regional flux. Such a mode of life could be further enhanced by abandoning movement as that could be seen as wasting precious energy. Hence, we hypothesise that when evolving in the presence of an energy model the sticky feet creatures will appear in a wider range of sizes during their evolution than happens in an 'unconstrained energy' world.

Similar hypotheses could be expressed about other physical aspects of the creatures. Here we explore just the size.

Results

Our simulations generate a large quantity of data and here we show just a single summary of one aspect of it. Figure 7 shows a plot of the inter-quartile range of the sizes (areas) of the population of creatures as it changes with the genome mutation distance. This figure includes data for three different configurations: the unconstrained 'control' situation, one with an energy flux of 80 (arbitrary) energy units, and one with an energy flux of 100 units. Data for this plot are taken from a total of over 40 separate simulation runs and summarise the simulated lives of over 350,000 energetic sticky feet creatures.

We have chosen these energy levels based on experience running our simulations. Below an energy flux of 80 units it is invariably the case that the population of creatures dies out. For example, in all our experimental data no creature has existed in a simulation with a flux of 70 with a higher
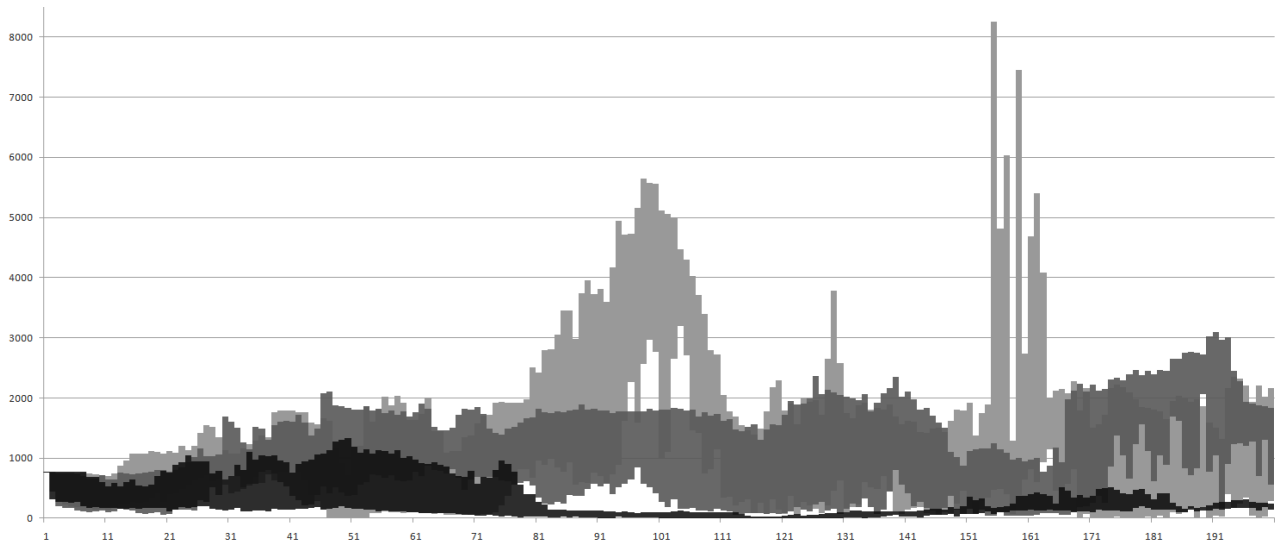
Figure 7: Summary of results of execution of energetic sticky feet simulation with mutation distance on the X axis. The foreground, darkest, distribution shows the inter-quartile range of the areas of sticky feet creatures across 200 mutation distances of evolution using the implementation that did not use an energy model. The mid-grey plot is the same information using the energy model at a flux level of 100 units. The pale grey plot shows the results using the energy model at a flux of 80 units.

mutation distance than 94. At a flux of 50, we see nothing beyond mutation distance 73.

Even without comparing the data with the unconstrained situation we see a clear effect of the flux on the simulated lives of the sticky feet creatures. Furthermore, inspection of figure 7 shows significant differences between the 'with energy' and 'unconstrained energy' variants of our simulation. For example, at mutation distance 200 (the largest we show on the figure) creatures in the *energy* = 80 world have a range of sizes from 280 area units at the lower quartile to 2170 units at the upper quartile. In the 'unconstrained energy' world the equivalent sizes are from 140 to 238 units.

Experience with our experiments, and observation of the results shown here, leads us to a further hypothesis. This is driven by the observation, seen in figure 7, that at *energy* = 100 there is less population diversity than at *energy* = 80. As we know that at lower energy levels the populations of sticky feet creatures usually dies out we hypothesise that there is a critical energy flux density, in a set of simulations with otherwise consistent parameters, that generates creature populations of the widest diversity. At low energy levels there is insufficient energy for populations to survive and hence they die out before generating significant diversity; at higher energy levels it becomes easier and easier to make a living, all the way up to the unconstrained world.

We choose a single statistic to represents diversity of simulations with a particular energy flux, and look to see if it varies in the hypothesised manner. The statistic we use is the range of sizes of creatures throughout all lifetimes at a particular energy level. Figure 8 is a box and whisker plot

of the interquartile range (IQR) of sizes of creatures over all mutation distances. In figure 8, the median represents the median IQR of sizes over mutation distance at a particular energy flux (the median size of the bars in figure 7): the larger the median, the larger the range of sizes, hence the greater the diversity. In figure 8, the IQR represents the variation in the IQR of sizes over mutation distance at a particular energy flux (the range of sizes of the bars in figure 7): the larger the IQR in figure 8, the larger the range of range of sizes, hence the greater the range of diversity. Observation of figure 8 does indeed show the hypothesised characteristic of a critical energy flux with maximum diversity.

Discussion

The hypotheses that we have discussed are supported by the experimental results we have included. Specifically, the results we see when running the 'energetic' simulations show a more diverse range of creatures being produced than in similar 'unconstrained energy' situations. Furthermore, there is a 'critical' energy level that supports the widest diversity. At lower energies we see less diverse populations that soon die out; at higher energies–which includes the unconstrained case–we see less diverse populations that nonetheless persist. The critical energy level is the point between a low energy world where eating other creatures is a necessity of life, but nevertheless there is not enough influx of energy to survive, and a high energy world where there is little evolutionary pressure, and sessile behaviour is common.

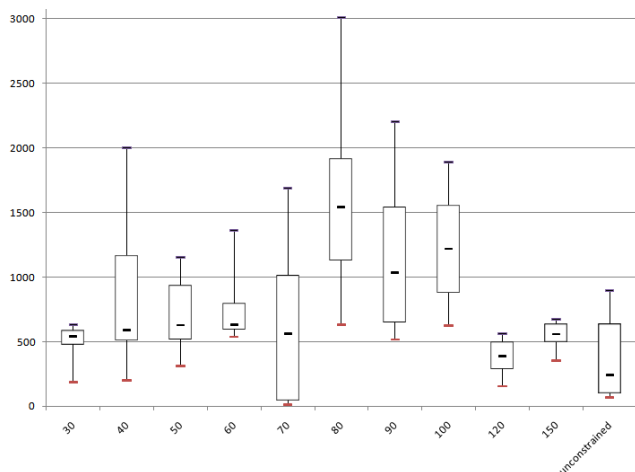Visual inspection of our simulations make it painfully

Figure 8: Box plot of inter-quartile ranges of creature sizes at various energy levels. Each plot shows the 9th percentile, the first quartile, the median, the third quartile and the 91st percentile for the distribution of the inter-quartile ranges at the given energy flux. The rightmost box is for the unconstrained energy version of the simulation.

clear that although we generate creatures with a wide range of sizes and structure they are still recognisably the same sort of thing: variations on a theme of feet and springs (figure 6). The end result is interesting but does not compare with biological evolution and the vast range of forms and structures that we see there. Our simulations could never generate such a range of structures because the creatures' representation, morphogenesis and mutation operators are fixed, even though the various probabilities of their application and effect may change. That is, although we have a general notion of the sorts of energy we are simulating, and this is encoded in our metamodel, we do not have a similar notion of a range of organisms. Therefore, the evolution we are exploring here is not fully open-ended. In order to do that we need a more abstract description of evolution.

Conclusions

Our metamodel summarises the essential components of an energy-rich world which is a basic feature of real world evolution, and also of artificial life. We have shown that application of this metamodel in even a simple manner yields more complex, more interesting, results.

However, our experiments also make it obvious that we need much more in order to approach real open-ended evolution. In particular we must be able to modify both the creatures and the kinds of modifications that the creatures undergo. Our current simulations do not support this.

Future Work

While interesting, our current simulation does not explore some aspects of the worlds implied by our metamodel. In particular we have not explored the notion of entropy, which we believe should open up further different ways of creatures making their living. We have also not explored a non-homogeneous world with, for example, a range of different energy fluxes and different levels of friction which could make, again, different modes of existence feasible.

And, as we have discussed, we would like to investigate ways of extending the kinds of evolution that occur in order to more closely approach true open-ended evolution.

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