Applied Biostatistics

Chi-squared tests

Martin Bland

Professor of Health Statistics

University of York

http://www-users.york.ac.uk/~mb55/

Analyses for qualitative data

Also called nominal, categorical.

Only two categories: dichotomous, attribute, quantal, binary.

Methods:

- Chi-squared test for association
- Fisher's exact test
- Chi-squared test for linear association
- Risk ratio, relative risk
- Odds ratio

Contingency tables

Cross tabulation of two categorical variables:

Time of delivery by housing tenure

Housing tenure	Premature	Term	Total
Owner-occupier	50	849	899
Council tenant	29	229	258
Private tenant	11	164	175
Lives with parents	6	66	72
Other	3	36	39
Total	99	1344	1443

This kind of cross-tabulation of frequencies is also called a **contingency table** or **cross classification**.

Want to test the null hypothesis that there is no relationship or association between the two variables.

Contingency tables

Time of delivery by housing tenure

Housing tenure	Premature	Term	Total
Owner-occupier	50	849	899
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Total	99	1344	1443

Want to test the null hypothesis that there is no relationship or association between the two variables.

If the sample is large, we can do this by a chi-squared test.

If the sample is small, we must use Fisher's exact test.

The chi-squared test for association

Time of delivery by housing tenure

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Null hypothesis: no association between the two variables.

Alternative hypothesis: an association of some type.

The chi-squared test for association

Time of delivery by housing tenure

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Owner-occupier			899
Council tenant			258
Private tenant			175
Lives with parents			72
Other			39
Total	99	1344	1443

Proportion who are premature = 99/1443

Out of 899 owner occupiers, expect $899 \times 99/1443 = 61.7$ to be premature deliveries if the null hypothesis were true.

The chi-squared test for association

Time of delivery by housing tenure

Housing tenure	Premature	Term	Total
Owner-occupier	61.7		899
Council tenant			258
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Council tenant			258
Private tenant			175
Lives with parents			72
Other			39
Total	99	1344	1443

Proportion who are term = 1344/1443

Out of 899 owner occupiers, expect 899 × 1344/1443 = 837.3 to be term deliveries if the null hypothesis were true.

The chi-squared test for association

Time of delivery by housing tenure

Housing tenure	Premature	Term	Total
Owner-occupier	61.7	837.3	899
Council tenant			258
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Proportion who are term = 1344/1443

Out of 899 owner occupiers, expect 899 × 1344/1443 = 837.3 to be term deliveries if the null hypothesis were true.

Note that 61.7 + 837.3 = 899.

The chi-squared test for association

Time of delivery by housing tenure

Housing tenure	Premature	Term	Total
Owner-occupier	61.7	837.3	899
Council tenant			258
Private tenant			175
Lives with parents			72
Other			39
Total	99	1344	1443

Out of 258 council tenants, expect 258 × 99/1443 = 17.7 to be premature deliveries if the null hypothesis were true.

Out of 258 council tenants, expect 258 × 1344/1443 = 240.3 to be term deliveries if the null hypothesis were true.

The chi-squared test for association

Time of delivery by housing tenure

Housing tenure	Premature	Term	Total
Owner-occupier	61.7	837.3	899
Council tenant	17.7	240.3	258
Private tenant			175
Lives with parents			72
Other			39
Total	99	1344	1443

Out of 258 council tenants, expect 258 × 99/1443 = 17.7 to be premature deliveries if the null hypothesis were true.

Out of 258 council tenants, expect 258 × 1344/1443 = 240.3 to be term deliveries if the null hypothesis were true.

The chi-squared test for association

Time of delivery by housing tenure

Housing tenure	Premature	Term	Total
Owner-occupier	61.7	837.3	899
Council tenant	17.7	240.3	258
Private tenant			175
Lives with parents			72
Other			39
Total	99	1344	1443

Out of 258 council tenants, expect 258 × 99/1443 = 17.7 to be premature deliveries if the null hypothesis were true.

Out of 258 council tenants, expect 258 × 1344/1443 = 240.3 to be term deliveries if the null hypothesis were true.

Note that 17.7 + 240.3 = 258.

The chi-squared test for association

Time of delivery by housing tenure

Housing tenure	Premature	Term	Total
Owner-occupier	61.7	837.3	899
Council tenant	17.7	240.3	258
Private tenant	12.0	163.0	175
Lives with parents			72
Other			39
Total	99	1344	1443

The chi-squared test for association

Time of delivery by housing tenure

Housing tenure	Premature	Term	Total
Owner-occupier	61.7	837.3	899
Council tenant	17.7	240.3	258
Private tenant	12.0	163.0	175
Lives with parents	4.9	67.1	72
Other			39
Total	99	1344	1443

The chi-squared test for association

Time of delivery by housing tenure

Housing tenure	Premature	Term	Total
Owner-occupier	61.7	837.3	899
Council tenant	17.7	240.3	258
Private tenant	12.0	163.0	175
Lives with parents	4.9	67.1	72
Other	2.7	36.3	39
Total	99	1344	1443

The chi-squared test for association

Time of delivery by housing tenure

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Note that 61.7 + 17.7 +12.0 + 4.9 + 2.7 = 99,

837.3 + 240.3 + 163.0 + 67.1 + 36.3 = 1344.

Observed and expected frequencies have the same row and column totals.

The chi-squared test for association

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Expected frequency if null hypothesis true =

$$\frac{\text{row total} \times \text{column total}}{\text{grand total}}$$

The chi-squared test for association

Time of delivery by housing tenure

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Owner-occupier	50	61.7	849	837.3	899
Council tenant	29	17.7	229	240.3	258
Private tenant	11	12.0	164	163.0	175
Lives with parents	6	4.9	66	67.1	72
Other	3	2.7	36	36.3	39
Total	99		1344		1443

Compare the observed and expected frequencies.

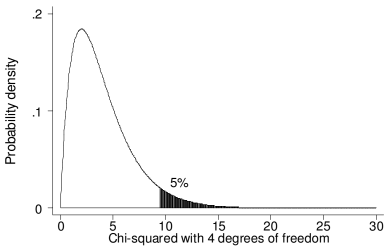
Add (observed – expected)^2/expected for all cells.

= 10.5.

If null hypothesis true and samples are large enough, this is an observation from a Chi-squared distribution, often written χ^2.

The Chi-squared distribution

Family of distributions, one parameter, called the **degrees of freedom**.



Percentage points of the Chi-squared Distribution

Degrees of freedom	Probability that the tabulated value is exceeded 10% 0.10	5% 0.05	1% 0.01	0.1% 0.001
1	2.71	3.84	6.63	10.83
2	4.61	5.99	9.21	13.82
3	6.25	7.81	11.34	16.27
4	7.78	9.49	13.28	18.47
5	9.24	11.07	15.09	20.52
6	10.64	12.59	16.81	22.46
7	12.02	14.07	18.48	24.32
8	13.36	15.51	20.09	26.13
9	14.68	16.92	21.67	27.88
10	15.99	18.31	23.21	29.59
.	.	.	.	.
.	.	.	.	.

The chi-squared test for association

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For a contingency table, the degrees of freedom are given by:

(number of rows – 1) × (number of columns – 1).

The chi-squared test for association

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For a contingency table, the degrees of freedom are given by:

(number of rows – 1) × (number of columns – 1).

We have (5 – 1) × (2 – 1) = 4 degrees of freedom.

The chi-squared test for association

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$\chi^2 = 10.5$, 4 d.f.

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	10% 0.10	5% 0.05	1% 0.01	0.1% 0.001
1	2.71	3.84	6.63	10.83
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(number of rows – 1) × (number of columns – 1).

We have (5 – 1) × (2 – 1) = 4 degrees of freedom.

χ^2 = 10.5, 4 d.f., P < 0.05.

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For a contingency table, the degrees of freedom are given by:

(number of rows – 1) × (number of columns – 1).

We have (5 – 1) × (2 – 1) = 4 degrees of freedom.

χ^2 = 10.5, 4 d.f., P < 0.05. Using a computer, P = 0.03.

The chi-squared test for association

The chi-squared statistic is not an index of the strength of the association.

If we double the frequencies, this will double chi-squared, but the strength of the association is unchanged.

The chi-squared test for association

The test statistic follows the Chi-squared Distribution provided the expected values are large enough.

This is a large sample test.

The smaller the expected values become, the more dubious will be the test.

The conventional criterion for the test to be valid is this: the chi-squared test is valid if at least 80% of the expected frequencies exceed 5 and all the expected frequencies exceed 1.

Also known as the **Pearson chi-squared test**.

Fisher's exact test

Also called the **Fisher-Irwin exact test**.

Works for any sample size.

Used to be used only for small samples in 2 by 2 tables, because of computing problems.

Calculate the probability of every possible table with the given row and column totals.

Sum the probabilities for all the tables as or less probable than the observed.

Fisher's exact test

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χ^2 = 10.5, 4 d.f. Using a computer, P = 0.033.

Fishers' exact test: P = 0.034.

Fisher's exact test

Renal failure and mortality in peritonitis patients (Fennell, unpublished)

Status after 3 months	Renal failure Yes	No	Total
Dead	6	31	37
Alive	2	62	64
Total	8	93	101

Fisher's exact test

Renal failure and mortality in peritonitis patients (Fennell, unpublished)

Status after 3 months	Renal failure Yes	No	Total
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Does not meet the chi-squared criterion.

Fisher's exact test

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Status after 3 months	Renal failure Yes		No	Total
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Alive	2		62	64
Total	8		93	101

Does not meet the chi-squared criterion.

Chi-squared = 5.51, df = 1, P = 0.019,

Fisher's exact test: P = 0.049.

Much bigger.

Yates' correction for 2 by 2 tables

Renal failure and mortality in peritonitis patients (Fennell, unpublished)

Status after 3 months	Renal failure Yes	No	Total
Dead	6	31	37
Alive	2	62	64
Total	8	93	101

Chi-squared = 5.51, df = 1, P = 0.019,

Fisher's exact test: P = 0.049.

Yates' chi-squared = 3.86, df = 1, P = 0.0495.

Very close to the Fisher probability.

The chi-squared test for linear association

Assessment of radiological appearance at six months as compared with appearance on admission (MRC 1948)

Radiological assessment	Streptomycin	Control
Considerable improvement	28	4
Moderate or slight improvement	10	13
No material change	2	3
Moderate or slight deterioration	5	12
Considerable deterioration	6	6
Deaths	4	14
Total	55	52

Chi-squared = 26.97, 5 d.f., P = 0.0001.

Does not take the ordering of the categories into account.

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Chi-squared = 26.97, 5 d.f., P = 0.0001.

Does not take the ordering of the categories into account.

Several tests do, including the Armitage chi-squared test for trend, the Mantel-Haenszel linear-by-linear association, Kendall’s tau b.

The chi-squared test for linear association

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Chi-squared = 26.97, 5 d.f., P = 0.0001.

Does not take the ordering of the categories into account.

SPSS does the Mantel-Haenszel linear-by-linear association chi-squared test, whether you want it or not.

The chi-squared test for linear association

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Chi-squared = 26.97, 5 d.f., P = 0.0001.

Does not take the ordering of the categories into account.

Trend: chi-squared = 17.93, 1 d.f., P < 0.0001.

Mantel-Haenszel linear-by-linear: chi-squared = 17.76, 1 d.f., P < 0.0001.

The chi-squared test for linear association

Mantel-Haenszel linear-by-linear: chi-squared = 17.76, 1 d.f., P < 0.0001.

Assigns numerical values to categories. This must be meaningful.

E.g.: Considerable improvement =1,
Moderate or slight improvement =2,
No material change = 3,
Moderate or slight deterioration = 4,
Considerable deterioration =5,
Death =6,

and

Streptomycin = 1,
Control =2.

The chi-squared test for linear association

Mantel-Haenszel linear-by-linear: chi-squared = 17.76, 1 d.f., P < 0.0001.

Assigns numerical values to categories. This must be meaningful.

We then say, given these numerical scales, is there a relationship of the form

improvement = constant + another constant × treatment

The chi-squared test for linear association

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Assigns numerical values to categories. This must be meaningful.

We then say, given these numerical scales, is there a relationship of the form

improvement = constant + another constant × treatment

Obviously, this cannot predict the improvement exactly.

Find the constants which give the best prediction for these data and then test whether they predict the improvement better than we would get by chance.

The chi-squared test for linear association

Mantel-Haenszel linear-by-linear: chi-squared = 17.76, 1 d.f., P < 0.0001.

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We then say, given these numerical scales, is there a relationship of the form

improvement = constant + another constant × treatment

Obviously, this cannot predict the improvement exactly.

Find the constants which give the best prediction for these data and then test whether they predict the improvement better than we would get by chance.

All we are interested is the test statistic, and we do not even see the two constants.

The chi-squared test for linear association

Mantel-Haenszel linear-by-linear: chi-squared = 17.76, 1 d.f., P < 0.0001.

Assigns numerical values to categories. This must be meaningful.

Should be valid even when the contingency chi-squared test is not, provided we have at least 30 observations.

Can be significant even when the contingency chi-squared is not.

It gives a more powerful test against a more restricted null hypothesis.

Applied Biostatistics

Chi-squared tests

Martin Bland
Professor of Health Statistics
University of York
http://www-users.york.ac.uk/~mb55/