Applied Biostatistics

Logarithms

Martin Bland

Professor of Health Statistics

University of York

http://www-users.york.ac.uk/~mb55/msc/

Logarithms

Mathematical function widely used in statistics.

$10^2 = 10\times10 = 100$	$\log_{10}(100) = 2$
$10^3 = 10\times10\times10 = 1000$	$\log_{10}(1000) = 3$
$10^5 = 10\times10\times10\times10\times10 = 100000$	$\log_{10}(100000) = 5$
$10^1 = 10$	$\log_{10}(10) = 1$

$\log_{10}(1000) + \log_{10}(100) = 3 + 2 = 5 = \log_{10}(100000)$

$1000 \times 100 = 100000$

Add on the log scale ➔ multiply on the natural scale.

$\log_{10}(1000) - \log_{10}(100) = 3 - 2 = 1 = \log_{10}(10)$

$1000 \div 100 = 10$

Subtract on the log scale ➔ divide on the natural scale.

Logarithms

$10^0 = 1$ $\log_{10}(1) = 0$

Why is this?

$\log_{10}(10) - \log_{10}(10) = 1 - 1 = 0$

$10 \div 10 = 1$

Logarithms do not have to be whole numbers.

$10^{0.5} = 10^{½} =$ root 10 = 3.1622777

We know this because $10^{½} \times 10^{½} = 10^{½+½} = 10^1 = 10$.

½ is the $\log_{10}$ of the square root of 10.

Logarithms

What is $\log_{10}(0)$?

It does not exist. There is no power to which we can raise 10 to give zero.

Logarithms of negative numbers do not exist, either.

We can only use logarithmic transformations for positive numbers.

Logarithms

If we multiple a logarithm by a number, on the natural scale we raise to the power of that number.

For example, $3\times\log_{10}(100) = 3\times2 = 6 = \log_{10}(1000000)$

and $100^3 = 1000000$.

If we divide a logarithm by a number, on the natural scale we take that number root.

For example, $\log_{10}(1000)/3 = 3/3 = 1 = \log_{10}(10)$

and the cube root of 1000 is 10, i.e. $10 \times 10 \times 10 = 1000$.

Logarithms

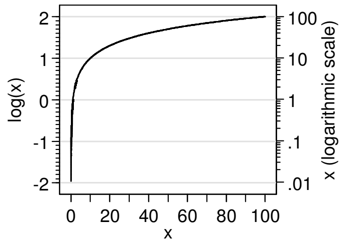
To convert from logarithms to the natural scale, we antilog.

$\text{antilog}_{10}(2) = 10^2 = 100$

On a calculator, use the 10^x key.

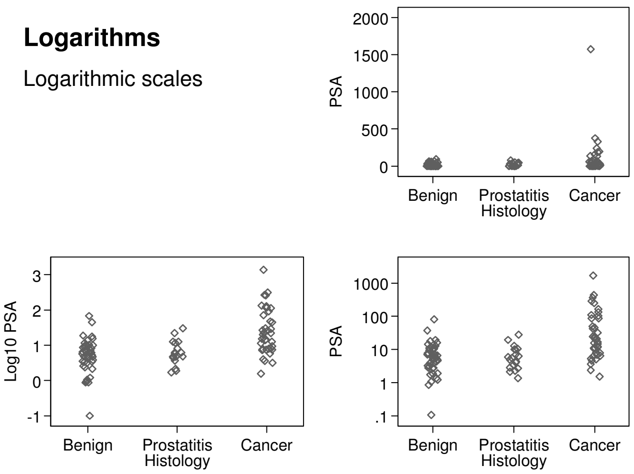
Logarithms

The logarithmic curve and logarithmic scale



Logarithms

Logarithmic scales



Logarithms

We can use logarithms to multiply or divide large numbers.

Logarithms to the base 10 are called **common logarithms**.

They were used for calculation before the age of cheap electronic calculators.

Mathematicians find it convenient to use a different base, called ‘e’, to give **natural logarithms**.

Logarithms

Mathematicians find it convenient to use a different base, called 'e', to give **natural logarithms**.

'e' is a number which cannot be written down exactly, like π.

e = 2.718281 . . .

They use this because the slope of the curve

$y = \log_{10}(x)$

is $\log_{10}(e)/x$. The slope of the curve

$y = \log_e(x)$

is $1/x$.

Using natural logs avoids awkward constants in formulae.

When you see 'log' written in statistics, it is the natural log.

Logarithms

To antilog from logs to base e on a calculator, use the key labelled 'e^x' or 'exp(x)'.