Applied Biostatistics

Proportions, risk ratios and odds ratios

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Risk difference

Cough during the day or at night at age 14 and bronchitis before age 5

Cough at age 14	Bronchitis before age 5 Yes		No		Total
Yes	26	9.5%	44	4.2%	70
No	247	90.5%	1002	95.8%	1249
Total	273	100.0%	1046	100.0%	1319

Want an estimate of the size of the bronchitis effect.

Difference between proportions:

0.095 – 0.042 = 0.053

or 9.5% – 4.2% = 5.3 percentage points.

Standard error for difference = 0.019, 95% CI:

0.053 – 1.96 × 0.019 to 0.053 + 1.96 × 0.019

= 0.016 to 0.090.

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Proportion who cough is called the **risk** of cough for that population.

Difference is **absolute risk difference**.

Risk ratio

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Proportion who cough is called the **risk** of cough for that population.

Difference is **absolute risk difference**.

Risk ratio = 0.095/0.042 = 2.26.

Also called **relative risk**, **RR**.

Risk ratio

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Risk ratio = 0.095/0.042 = 2.26.

Because risk ratio is a ratio, it has a very awkward distribution.

If we take the log of the risk ratio, we have something which is found by adding and subtracting log frequencies.

The distribution becomes approximately Normal.

Provided frequencies are not small, simple standard error.

Risk ratio

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Risk ratio = 0.095/0.042 = 2.26.

$\log_e(RR) = 0.817$.

SE for $\log_e(RR) = 0.238$.

95% CI for $\log_e(RR)$
$= 0.817 - 1.96 \times 0.238$ to $0.817 + 1.96 \times 0.238$
$= 0.351$ to 3.607.

95% CI for RR = exp(0.351) to exp(1.283) = 1.42 to 3.61.

Risk ratio

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log(RR) = 0.817, 95% CI = 0.351 to 1.283.

Risk ratio = 2.26, 95% CI = 1.42 to 3.61.

RR is not in the middle of its confidence interval.

The interval is symmetrical on the log scale, not the natural scale.

Odds

	Cough		No cough		Total
Bronchitis	26	9.5%	247	90.5%	273 100%

Risk of cough = 26/273 = 0.095

Odds of cough = 26/247 = 0.105

Risk = number experiencing event divided by number who could.

Odds = number experiencing event divided by number who did not experience event.

Odds

	Cough		No cough		Total
Bronchitis	26	9.5%	247	90.5%	273 100%

Risk of cough = 26/273 = 0.095

Odds of cough = 26/247 = 0.105

Risk: for every child, 0.095 children cough,
for every 100 children, 9.5 children cough.

Odds: for every child who does not cough, 0.105 children cough,
for every 100 children who do not cough, 10.5 children cough.

Odds ratio

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Odds of cough given bronchitis: 26/247 = 0.105.

Odds of cough given no bronchitis: 44/1002 = 0.044.

Odds ratio = (26/247)/(44/1002) = 0.105/0.044 = 2.397.

For every child who does not cough, 2.397 times as many will cough with a history bronchitis as will cough with no history of bronchitis.

Odds ratio

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Odds ratio, OR = (26/247)/(44/1002) = 2.397.

Like RR, OR has an awkward distribution. We use the log odds ratio.

The distribution becomes approximately Normal.

Provided frequencies are not small, simple standard error.

Odds ratio

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Odds ratio, OR = (26/247)/(44/1002) = 2.397.

$\log_e(OR) = 0.874$

SE $\log_e(OR) = 0.257$

95% CI for $\log_e(OR)$
$= 0.874 - 1.96 \times 0.257$ to $0.874 + 1.96 \times 0.257$
$= 0.370$ to 1.379.

95% CI for OR = exp(0.370) to exp(1.379) = 1.45 to 3.97.

Odds ratio

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$\log_e$(OR) = 0.874, 95% CI = 0.370 to 1.379.

OR = 2.397, 95% CI = 1.45 to 3.97.

OR is not in the middle of its confidence interval.

The interval is symmetrical on the log scale, not the natural scale.

Odds ratio

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Odds ratio for cough = (26/247)/(44/1002) = 2.397.

Doesn't matter which way round we do it.

Odds ratio for bronchitis =(26/44)/(247/1002) = 2.397.

Both OR = (26×1002)/(44×247) = 2.397.

Ratio of cross products.

Not true for relative risk.

Odds ratio

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Switching the rows or columns inverts the odds ratio.

Odds ratio for no cough given a history of bronchitis:
OR = (247/26)/(1002/44) = 0.417 = 1/2.397.

There are only two possible odds ratios.

On the log scale, equal and opposite.

$\log_e(2.397) = 0.874$, $\log_e(0.417) = -0.874$.