Health Sciences M.Sc. Programme

Applied Biostatistics

Week 6: Proportions, risk ratios and odds ratios

Risk ratio or relative risk

Chi-squared tests are tests of significance, they do not provide estimates of the strength of relationships. There are different ways of doing this for different kinds of data and sizes of table, but two are particularly important in health research: the risk ratio or relative risk and the odds ratio. Both apply to tables with two rows and columns.

Table 1. Cough during the day or at night at age 14 and bronchitis before age 5

Cough at age 14	Bronchitis at age 5		Total
	Yes	No	
Yes	26	44	70
No	247	1002	1249
Total	273	1046	1319

For example, consider Table 1. We ask: do children with bronchitis in infancy get more respiratory symptoms in later life than others? We have 273 children with a history of bronchitis before age 5 years, 26 of whom were reported to have day or night cough at the age 14. We have 1046 children with no bronchitis before age 5 years, 44 of whom were reported to have day or night cough at age 14. We thus have two proportions, 26/273 = 0.095 = 9.5% and 44/1046 = 0.042 = 4.2%.

We can find the difference between them, 0.095 – 0.042 = 0.053. We can find a standard error for this, 0.019, and use it to calculate a 95% confidence interval for the difference:
0.053 – 1.96 × 0.019 to 0.053 + 1.96 × 0.019 = 0.016 to 0.090. We could also write this difference as 9.5% – 4.2% = 5.3. We call this difference between two percentages 5.3 **percentage points** rather than a percentage.

The proportion who cough is called the **risk** of cough for that population. Thus in Table 1 the risk that a child with bronchitis before age 5 will cough at age 14 is 26/273 = 0.095, and the risk for a child without bronchitis before age 5 is 44/1046 = 0.042. The difference is called the **absolute risk difference**.

This difference in proportions may not very easy to interpret. The ratio of two proportions is often more useful. The ratio of the proportion with cough at age 14 for bronchitis before 5 to the proportion with cough at age 14 for those without bronchitis before 5 is 0.095/0.042 = 2.26. Children with bronchitis before 5 are more than twice as likely to cough during the day or at night at age 14 than children with no such history. We call this ratio the **risk ratio** or **relative risk**. Both can be abbreviated to **RR**.

The standard error for this ratio is complex, and as it is a ratio rather than a difference it does not approximate well to a Normal distribution. If we take the logarithm of the ratio, however, we get a difference, because the logarithm of a ratio is the difference between the two logarithms. We can find the standard error for the log ratio quite easily. For the example the log ratio = 0.817 and the standard error = 0.238. The 95% confidence interval for the log ratio is therefore
0.817 – 1.96 × 0.238 to 0.817 + 1.96 × 0.238 = 0.351 to 1.283. The 95% confidence interval for the ratio of proportions itself is the antilog of this, 1.42 to 3.61. Thus we estimate that the

proportion of children reported to cough during the day or at night among those with a history of bronchitis is between 1.4 to 3.6 times the proportion among those without a history of bronchitis.

The odds ratio

The probability or risk of an event is the number experiencing the event divided by the number who could experience it. The odds of an event is the number experiencing the event divided by the number who do not experience it. For example, the risk of cough for a child with a history of bronchitis = 26/273, the odds of cough for a child with a history of bronchitis = 26/247.

In terms of risk, for every child, 0.095 children cough, for every 100 children, 9.5 children cough. In terms of odds, for every child who does not cough, 0.105 children cough, for every 100 children who do not cough, 10.5 children cough.

Consider Table 1. The probability of cough for children with a history of bronchitis is 26/273 = 0.095. The odds of cough for children with a history of bronchitis is 26/247 = 0.105. The probability of cough for children without a history of bronchitis is 44/1046 = 0.042. The odds of cough for children without a history of bronchitis is 44/1002 = 0.044.

One way to compare children with and without bronchitis is to find the difference between the proportions of children with cough in the two groups. Another is to find the **odds ratio**, often abbreviated to **OR**, the ratio of the odds of cough in children with bronchitis and children without bronchitis. This is (26/247)/(44/1002) = 0.105/0.044 = 2.40. Thus the odds of cough in children with a history of bronchitis is 2.40 times the odds of cough in children without a history of bronchitis. This is not the same as the relative risk.

Like RR, OR has an awkward distribution. We use the log odds ratio, which will follow an approximately Normal distribution provided the frequencies are not too small, and has a simple standard error. Hence we can find the 95% confidence interval. For Table 1, the log odds ratio = 0.874, with standard error = 0.257. Hence the approximate 95% confidence interval is $0.874 - 1.96 \times 0.257$ to $0.874 + 1.96 \times 0.257 = 0.370$ to 1.379. To get a confidence interval for the odds ratio itself we must antilog, giving 1.45 to 3.97.

The odds ratio for cough given a history of bronchitis = (26/247)/(44/1002) = 2.397. The odds ratio for a history of bronchitis given a current cough = (26/44)/(247/1002) = 2.397. It doesn't matter which way round we do it. Both OR = (26×1002)/(44×247) = 2.397. We therefore also call the odds ratio the **ratio of cross products**. This is not true for relative risk.

Switching the rows or columns inverts the odds ratio. For example, the odds ratio for no cough given a history of bronchitis = (247/26)/(1002/44) = 0.417 = 1/2.397. This is the reciprocal of the OR for cough. There are only two possible odds ratios, as switching both rows and columns gives us odds ratio we started with. On the log scale, these two odds ratios are equal and opposite: $\log_e(2.397) = 0.874$, $\log_e(0.417) = -0.874$.

J. M. Bland,
17 August 2006