

**Clinical Biostatistics**

**Correlation**

Martin Bland

Professor of Health Statistics

University of York

<http://martinbland.co.uk/>

---

---

---

---

---

---

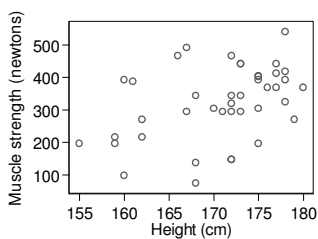
---

---

**Correlation**

Example: Muscle strength and height in 42 alcoholics

A scatter diagram:



How close is the relationship?

Correlation: measures closeness to a linear relationship.

---

---

---

---

---

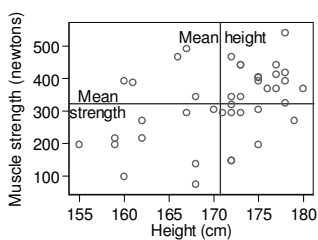
---

---

---

**Correlation coefficient**

Subtract means from observations and multiply.



Sum of products about the means.

Like the sum of squares about the means used for measuring variability.

---

---

---

---

---

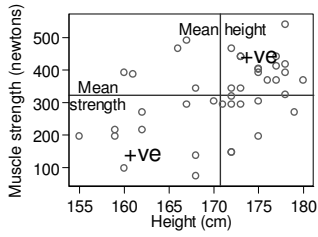
---

---

---

### Correlation coefficient

Subtract means from observations and multiply.



Products in top right and bottom left quadrants positive.

---

---

---

---

---

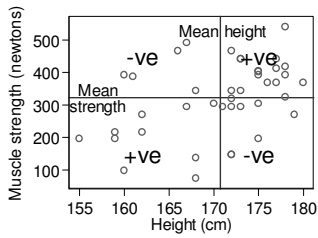
---

---

---

### Correlation coefficient

Subtract means from observations and multiply.



Products in top right and bottom left quadrants positive.  
Products in top left and bottom right quadrants negative.

---

---

---

---

---

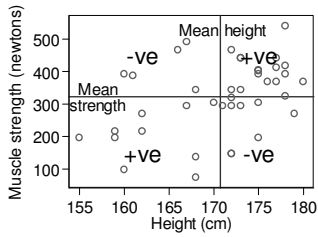
---

---

---

### Correlation coefficient

Subtract means from observations and multiply.



Sum of products positive.  
Correlation positive.

---

---

---

---

---

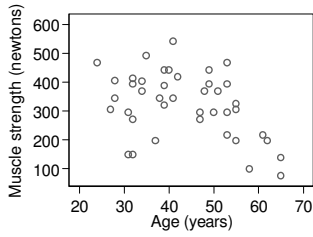
---

---

---

### Correlation coefficient

Example: Muscle strength and age in 42 alcoholics



---

---

---

---

---

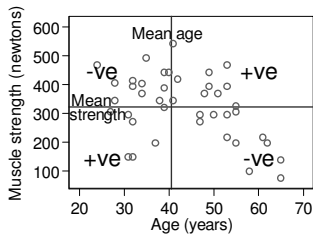
---

---

---

### Correlation coefficient

Example: Muscle strength and age in 42 alcoholics



Sum of products negative.  
Correlation negative.

---

---

---

---

---

---

---

---

### Correlation coefficient

Divide sum of products by square roots of sums of squares.

Correlation coefficient, denoted by  $r$ .

Maximum value = 1.00.

Minimum value = -1.00.

Also known as:

- Pearson's correlation coefficient,
- product moment correlation coefficient.

---

---

---

---

---

---

---

---

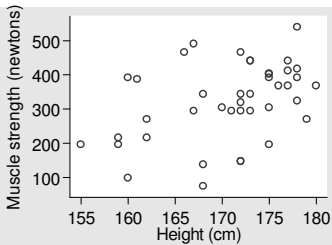
### Correlation coefficient

Divide sum of products by square roots of sums of squares.

Correlation coefficient, denoted by  $r$ .

Maximum value = 1.00.

Minimum value = -1.00.



---

---

---

---

---

---

---

---

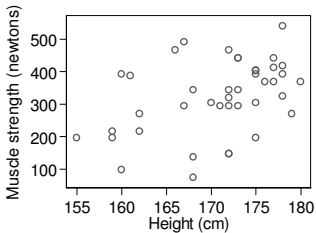
### Correlation coefficient

Divide sum of products by square roots of sums of squares.

Correlation coefficient, denoted by  $r$ .

Maximum value = 1.00.

Minimum value = -1.00.



$r = 0.42$ .  
Positive correlation of fairly low strength

---

---

---

---

---

---

---

---

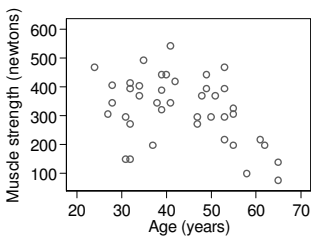
### Correlation coefficient

Divide sum of products by square roots of sums of squares.

Correlation coefficient, denoted by  $r$ .

Maximum value = 1.00.

Minimum value = -1.00.



$r = -0.42$ .  
Negative correlation of fairly low strength.

---

---

---

---

---

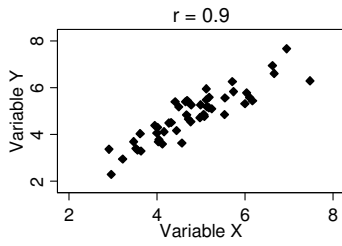
---

---

---

### Correlation coefficient

Positive when large values of one variable are associated with large values of the other.



---

---

---

---

---

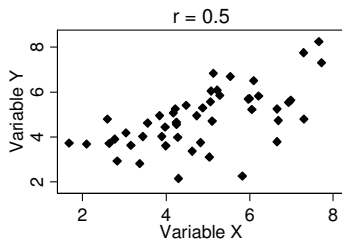
---

---

---

### Correlation coefficient

Positive when large values of one variable are associated with large values of the other.



---

---

---

---

---

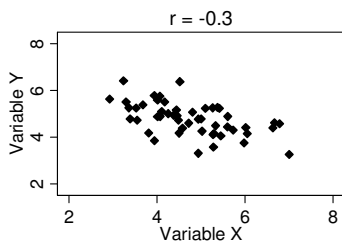
---

---

---

### Correlation coefficient

Negative when large values of one variable are associated with small values of the other.



---

---

---

---

---

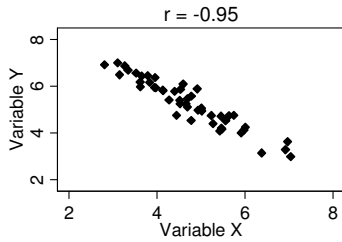
---

---

---

### Correlation coefficient

Negative when large values of one variable are associated with small values of the other.



---

---

---

---

---

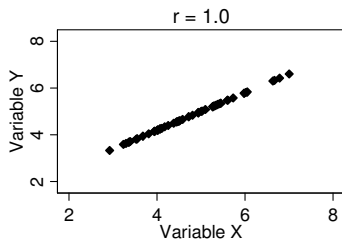
---

---

---

### Correlation coefficient

$r = +1.00$  when large values of one variable are associated with large values of the other and the points lie on a straight line.



---

---

---

---

---

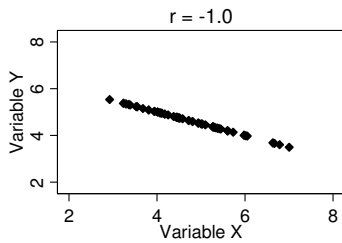
---

---

---

### Correlation coefficient

$r = -1.00$  when large values of one variable are associated with small values of the other and the points lie on a straight line.



---

---

---

---

---

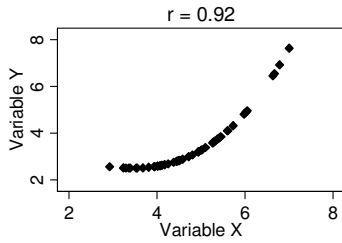
---

---

---

### Correlation coefficient

$r$  will not equal  $-1.00$  or  $+1.00$  when there is a perfect relationship unless the points lie on a straight line.



---

---

---

---

---

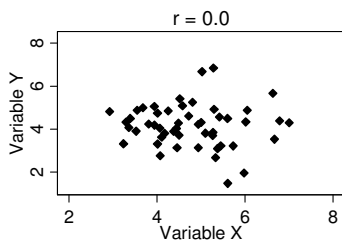
---

---

---

### Correlation coefficient

$r = 0.00$  when there is no linear relationship.



---

---

---

---

---

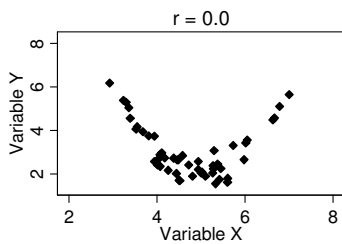
---

---

---

### Correlation coefficient

It is possible for  $r$  to be equal to  $0.00$  when there is a relationship which is not linear.



---

---

---

---

---

---

---

---

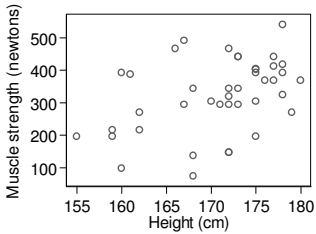
### Correlation coefficient

We can test the null hypothesis that the correlation coefficient in the population is zero.

Simple t test, tabulated.

Assume: one of the variables is from a Normal distribution.

Large deviations from assumption → P very unreliable.



$r = 0.42$ ,  $P = 0.006$ .

Easy to do, simple tables.

Computer programs almost always print this.

---

---

---

---

---

---

---

---

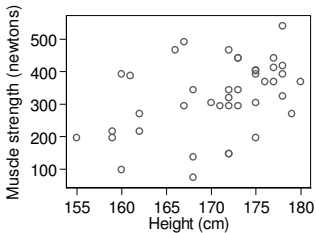
### Correlation coefficient

We can find a confidence interval for the correlation coefficient in the population.

Fisher's z transformation.

Assume: both of the variables are from a Normal distribution.

Large deviations from assumption → CI very unreliable.



$r = 0.42$ , approximate 95% confidence interval: 0.13 to 0.64

Tricky, approximate.

Computer programs rarely print this.

---

---

---

---

---

---

---

---