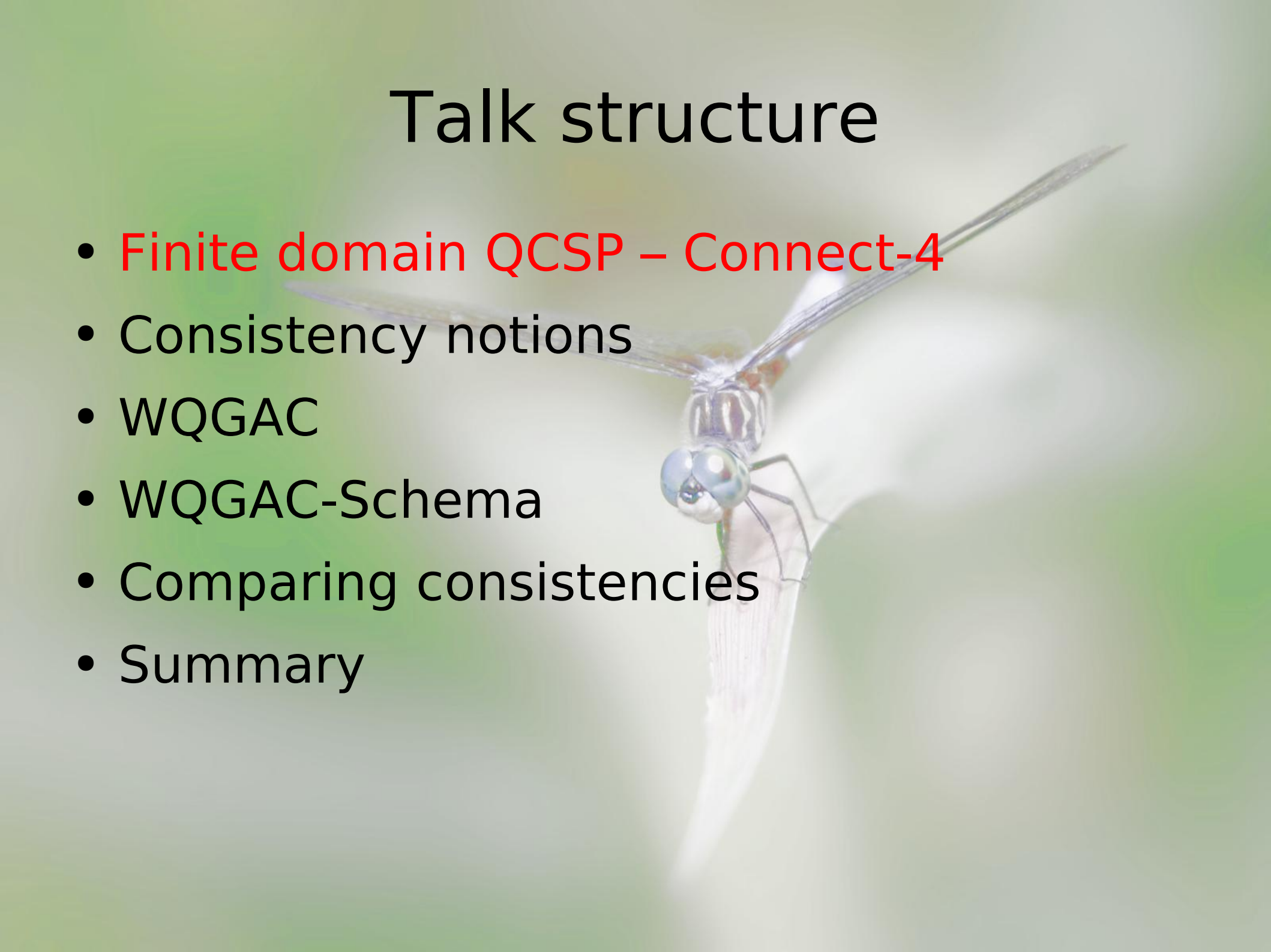
Consistency for Quantified Constraint Satisfaction Problems

Peter Nightingale

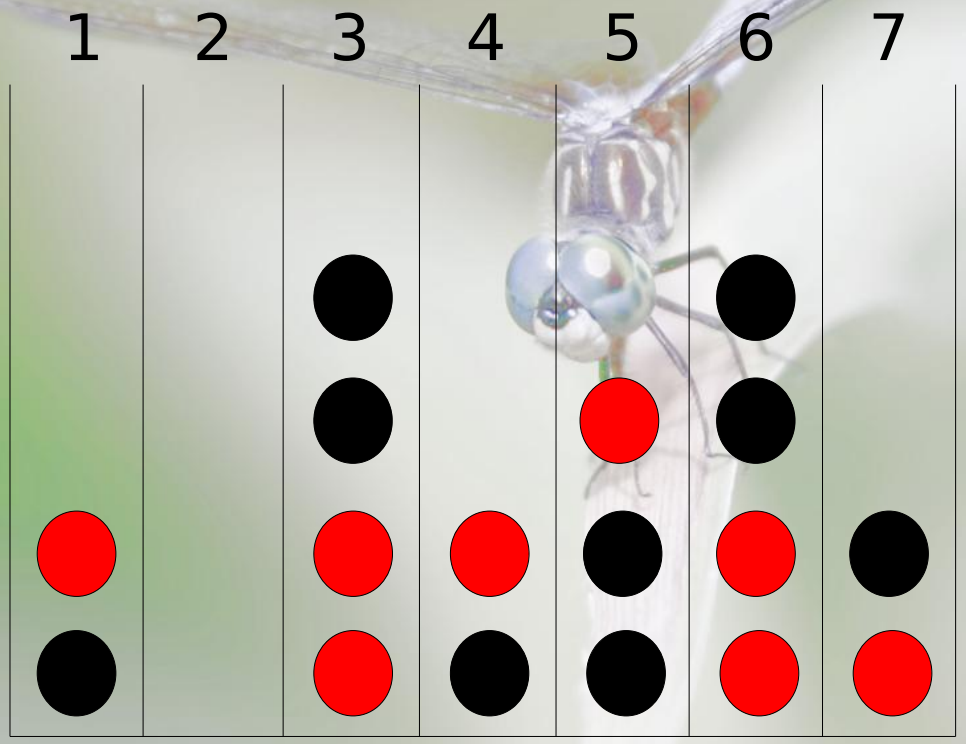
Talk structure

- Finite domain QCSP – Connect-4
- Consistency notions
- WQGAC
- WQGAC-Schema
- Comparing consistencies
- Summary



Finite domain QCSP

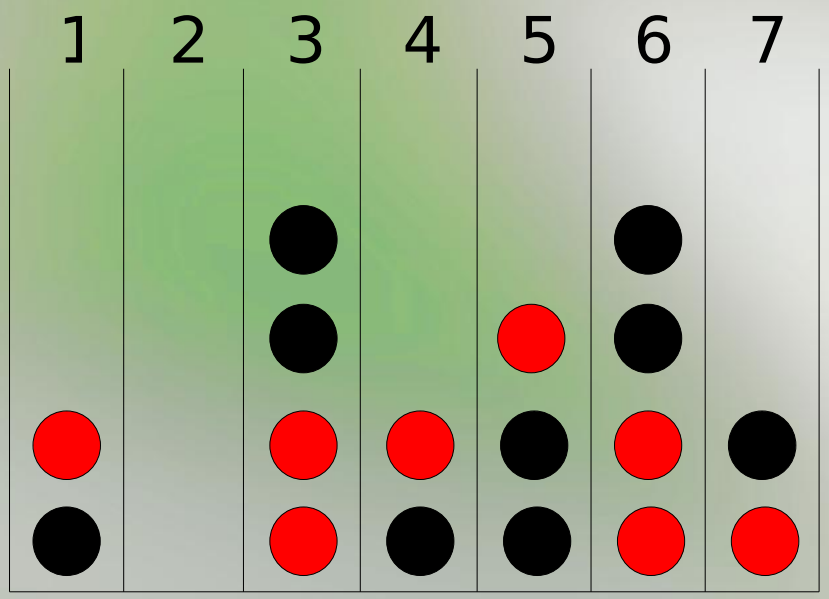
- Connect-4 endgame

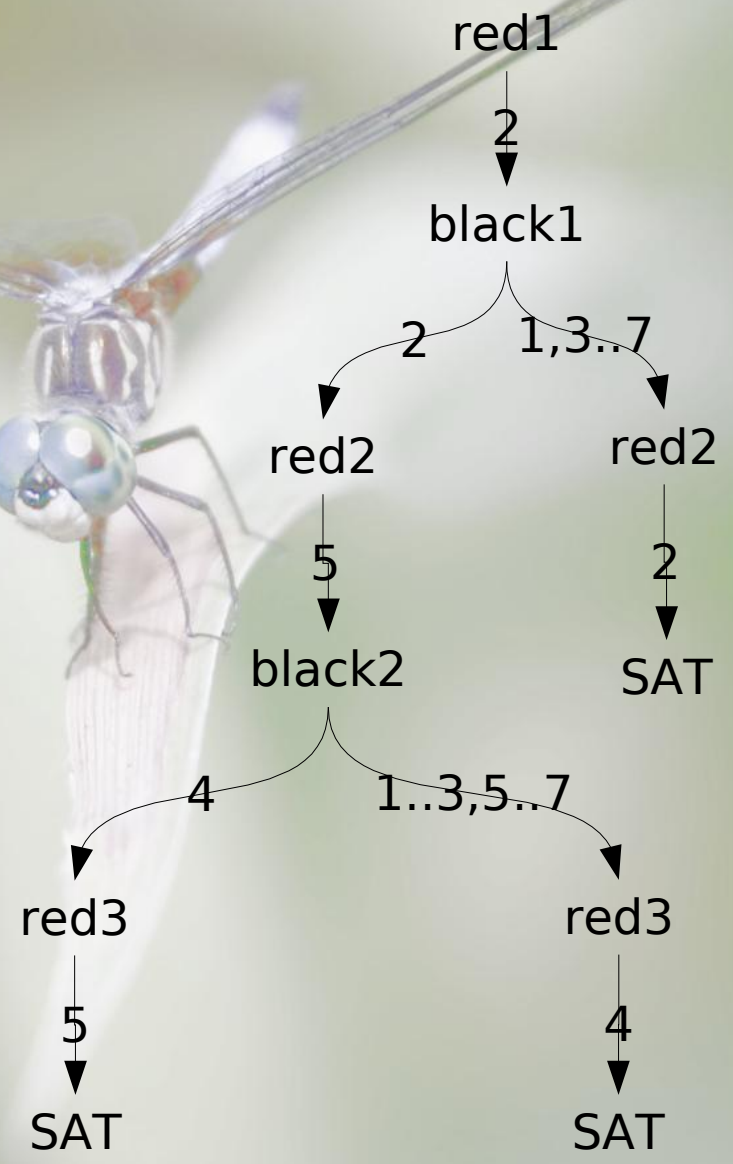


$$\exists red1 \forall black1 \exists red2 \forall black2 \exists red3 : redwins(red1, black1, red2, black2, red3)$$

Finite domain QCSP

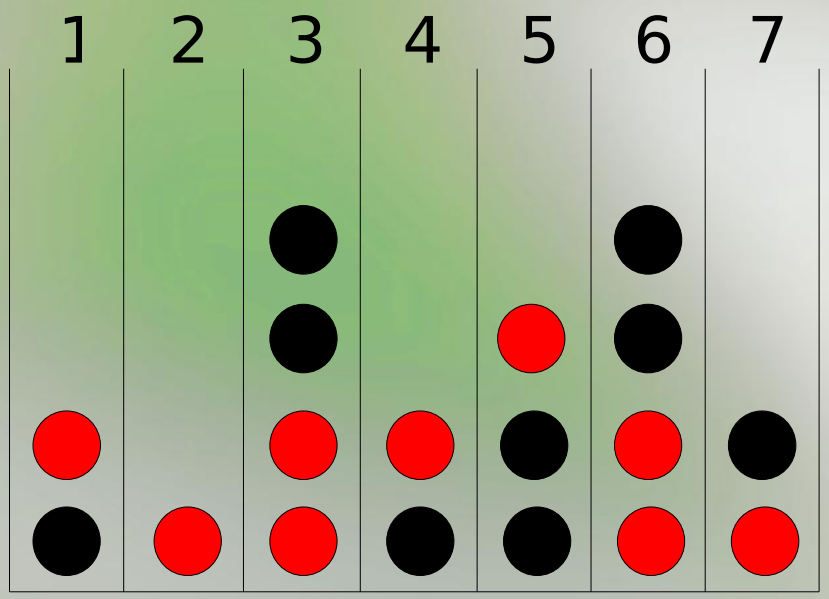
- Example strategy

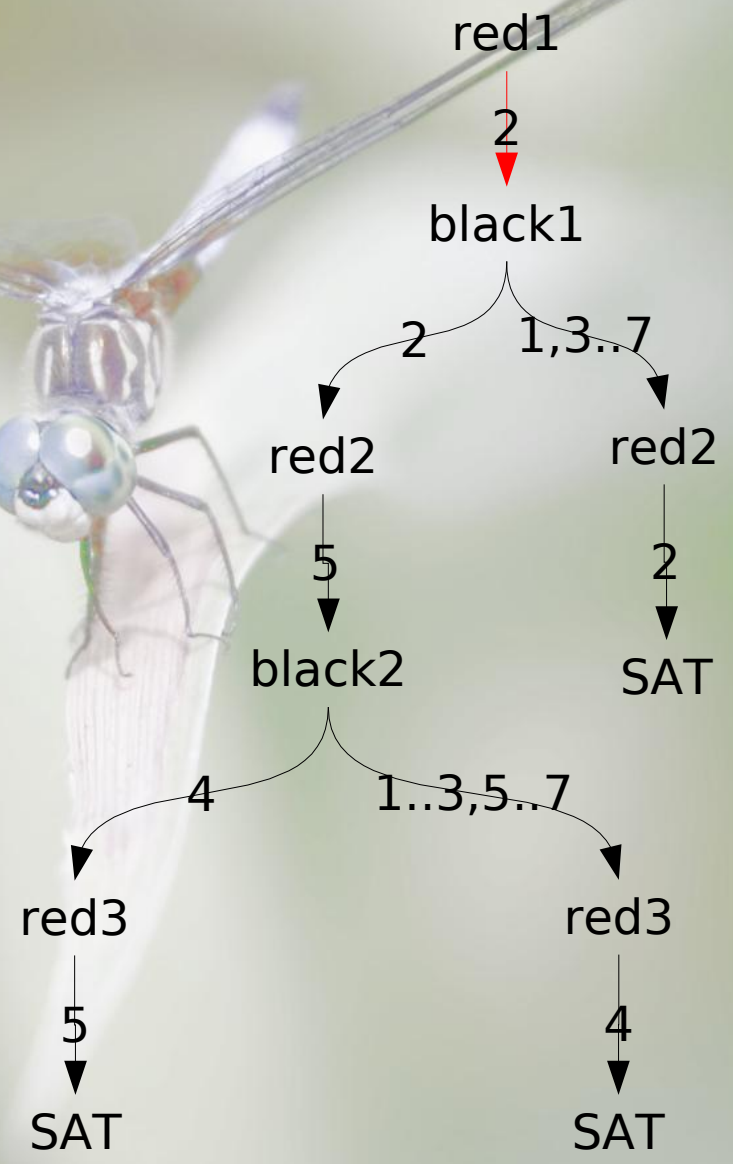




Finite domain QCSP

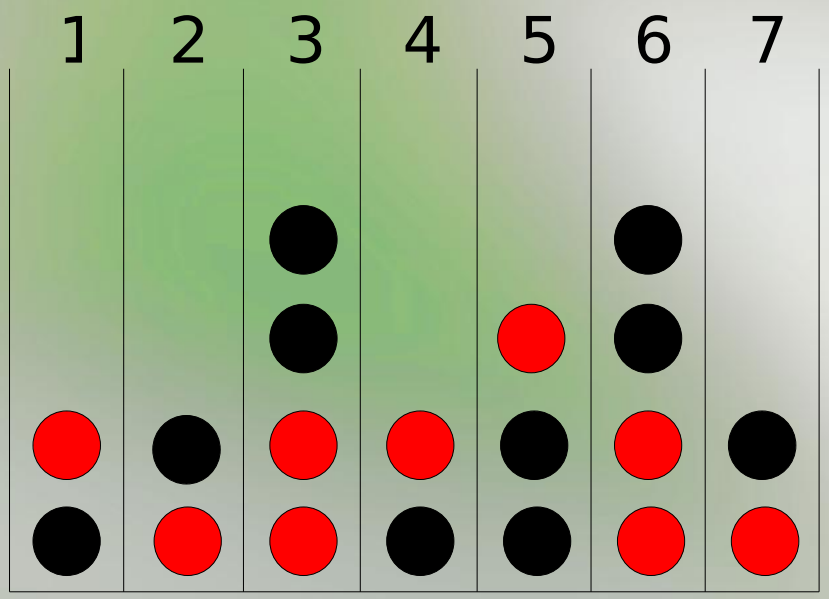
- Example strategy

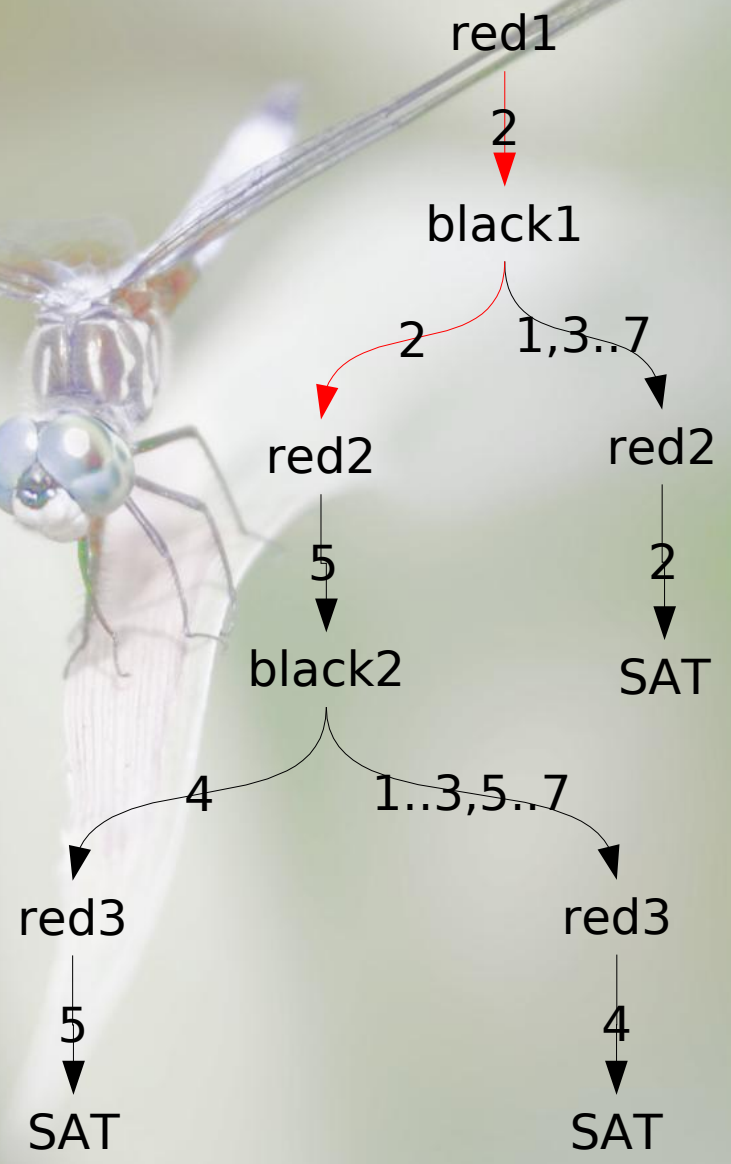




Finite domain QCSP

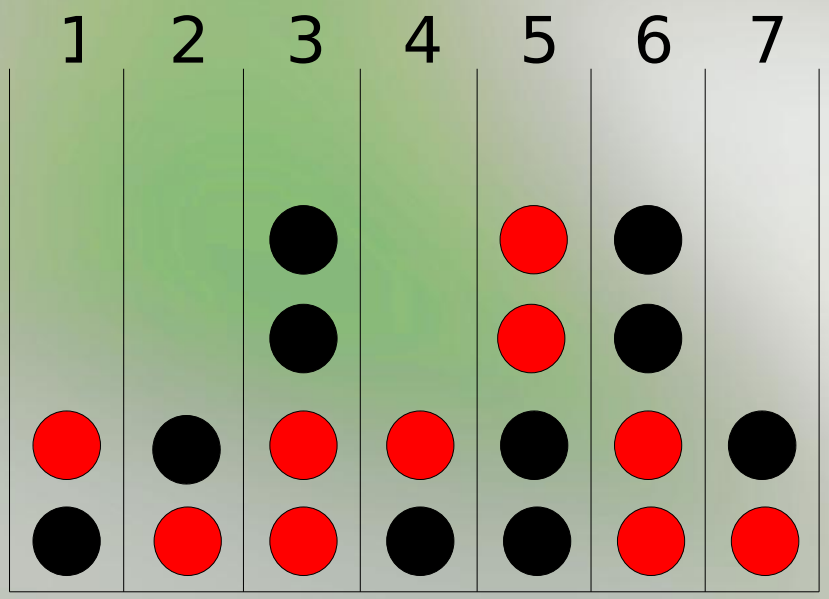
- Example strategy

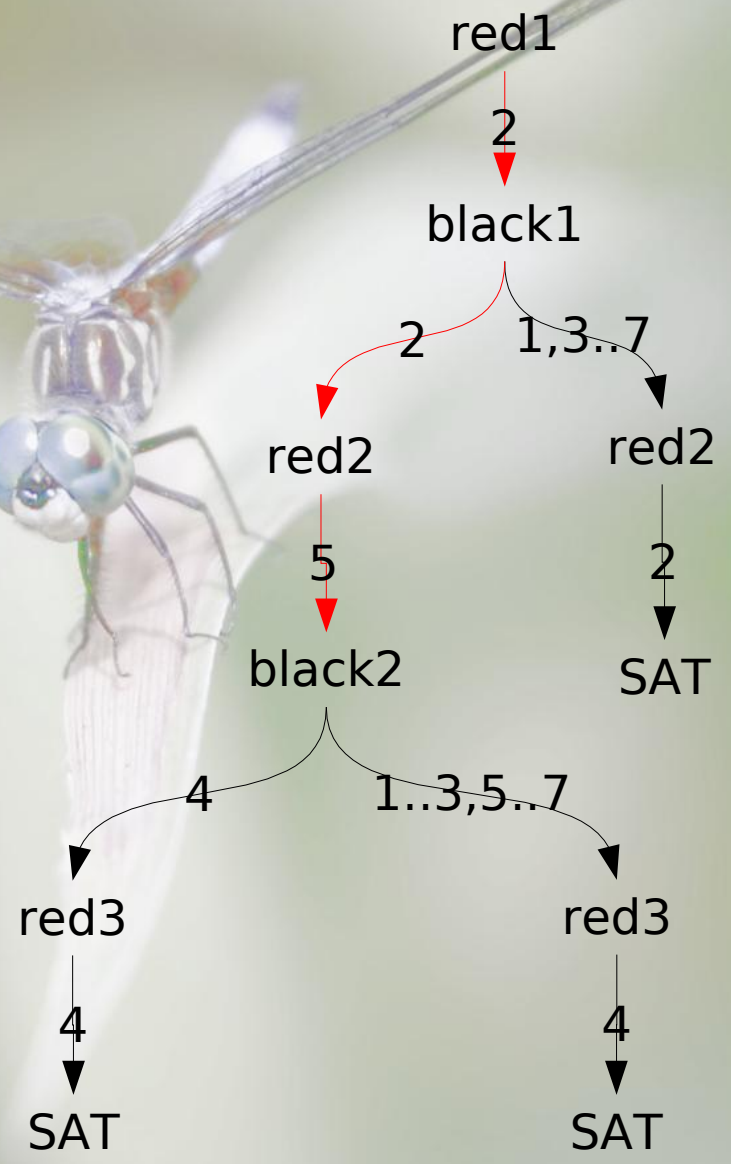




Finite domain QCSP

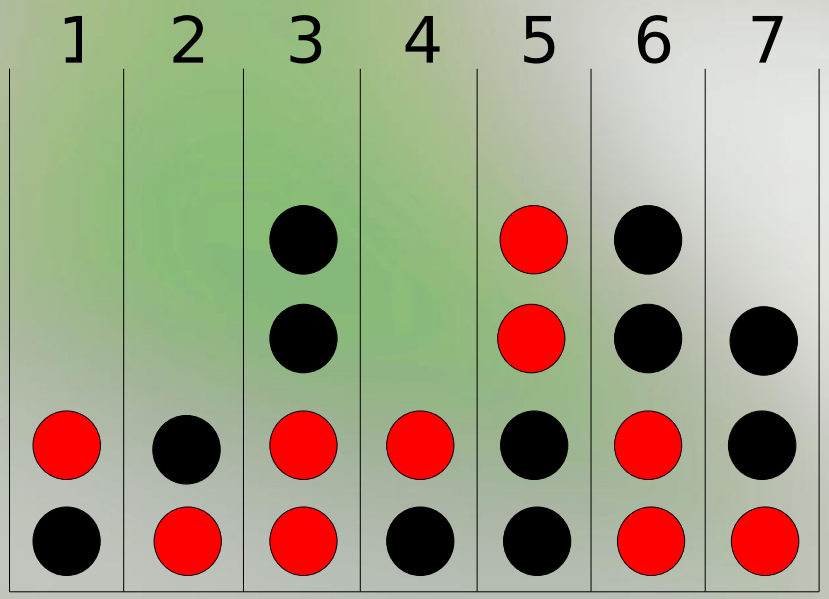
- Example strategy

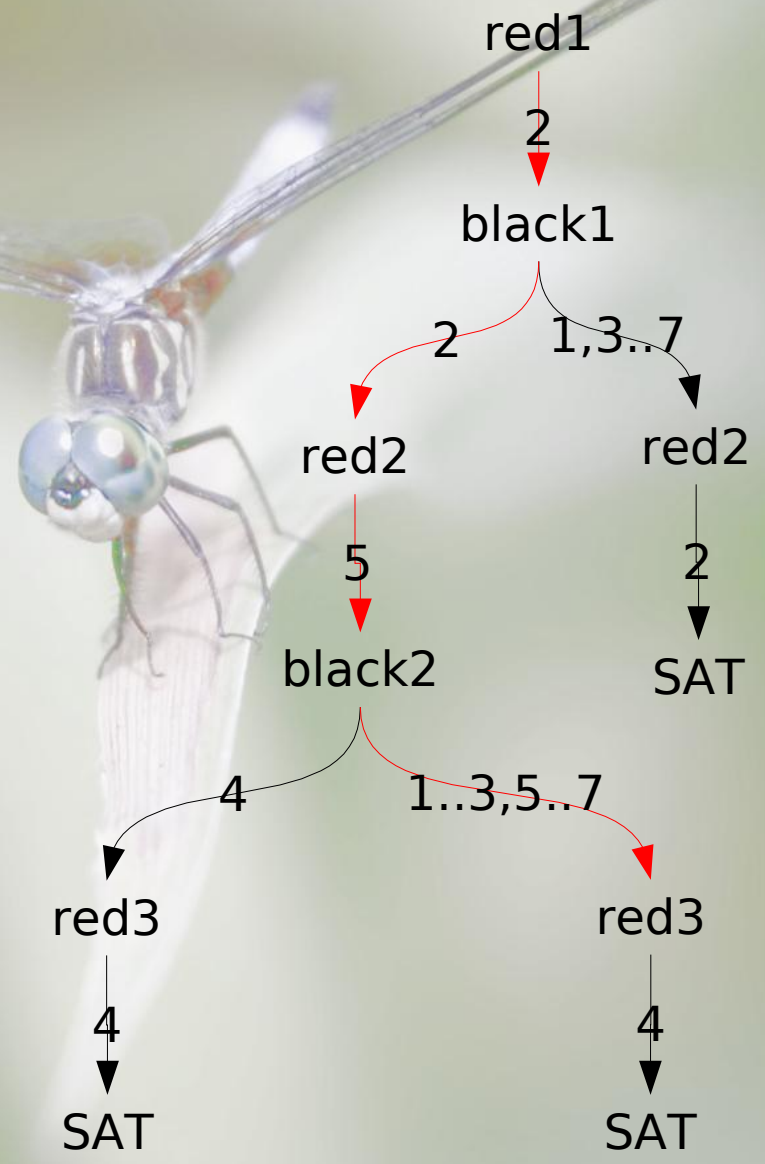




Finite domain QCSP

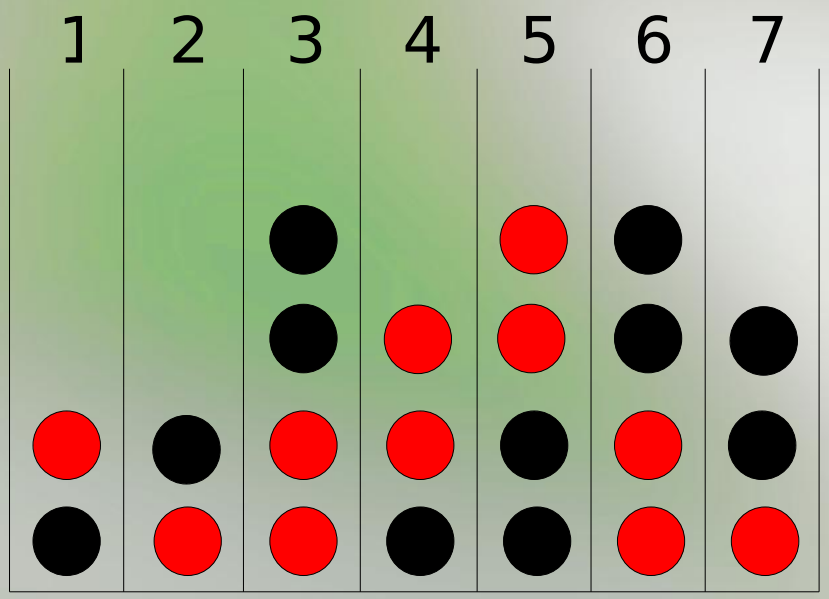
- Example strategy

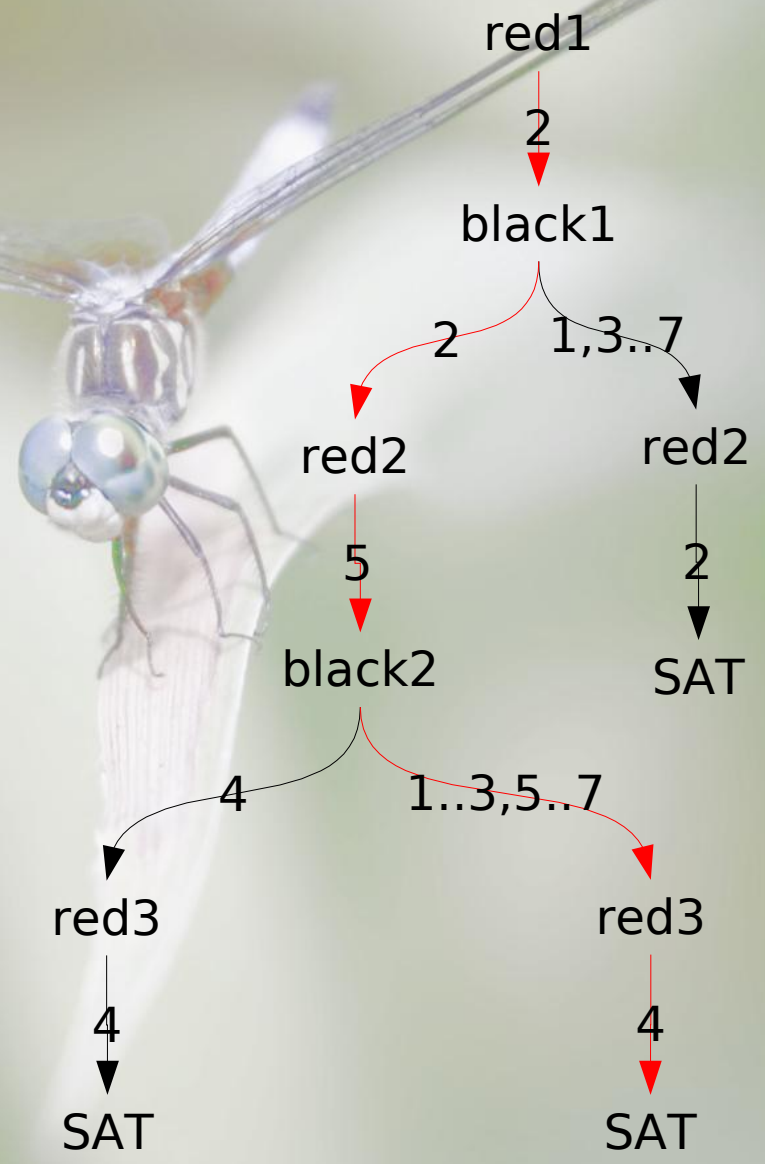




Finite domain QCSP

- Example strategy





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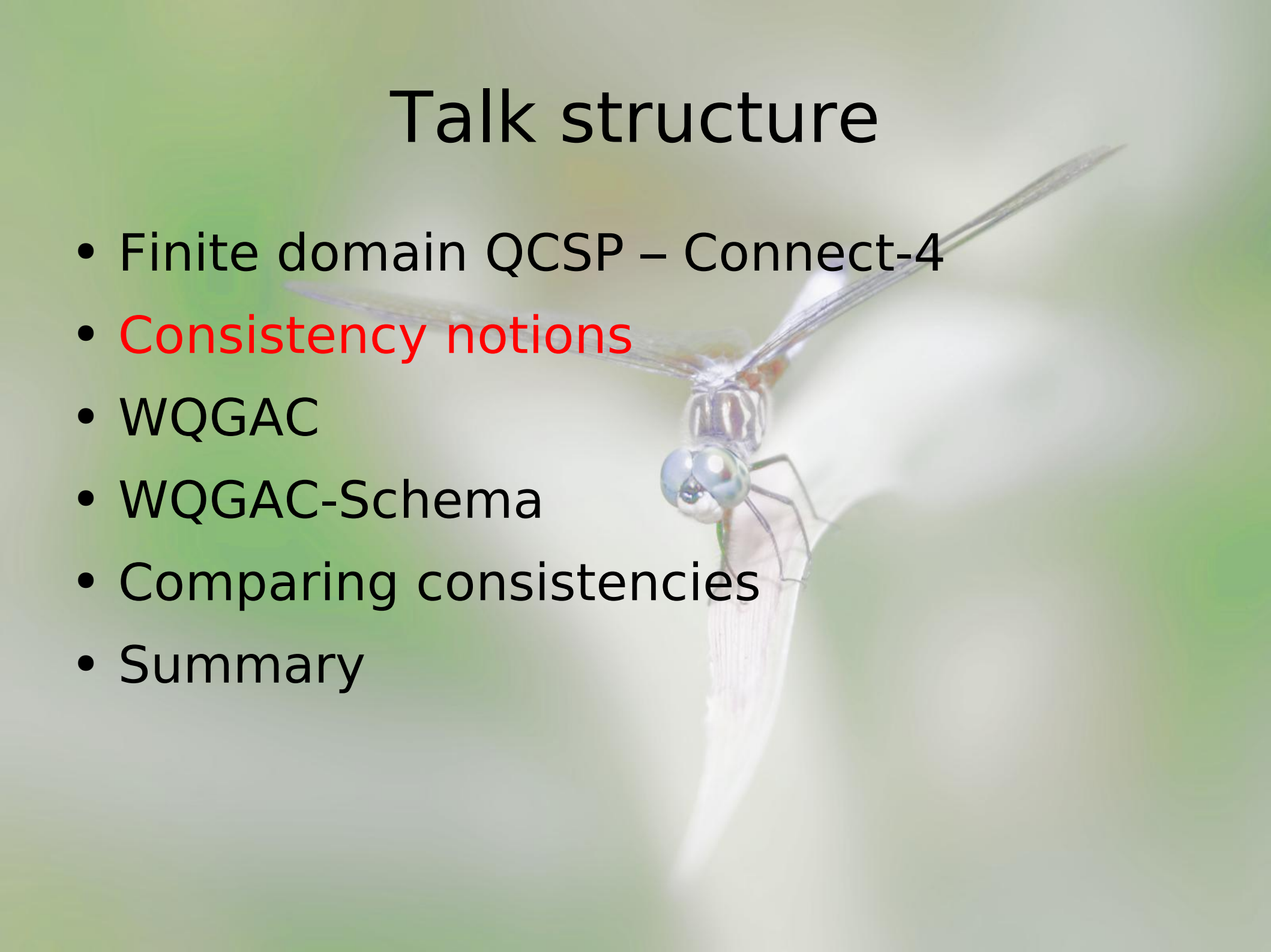
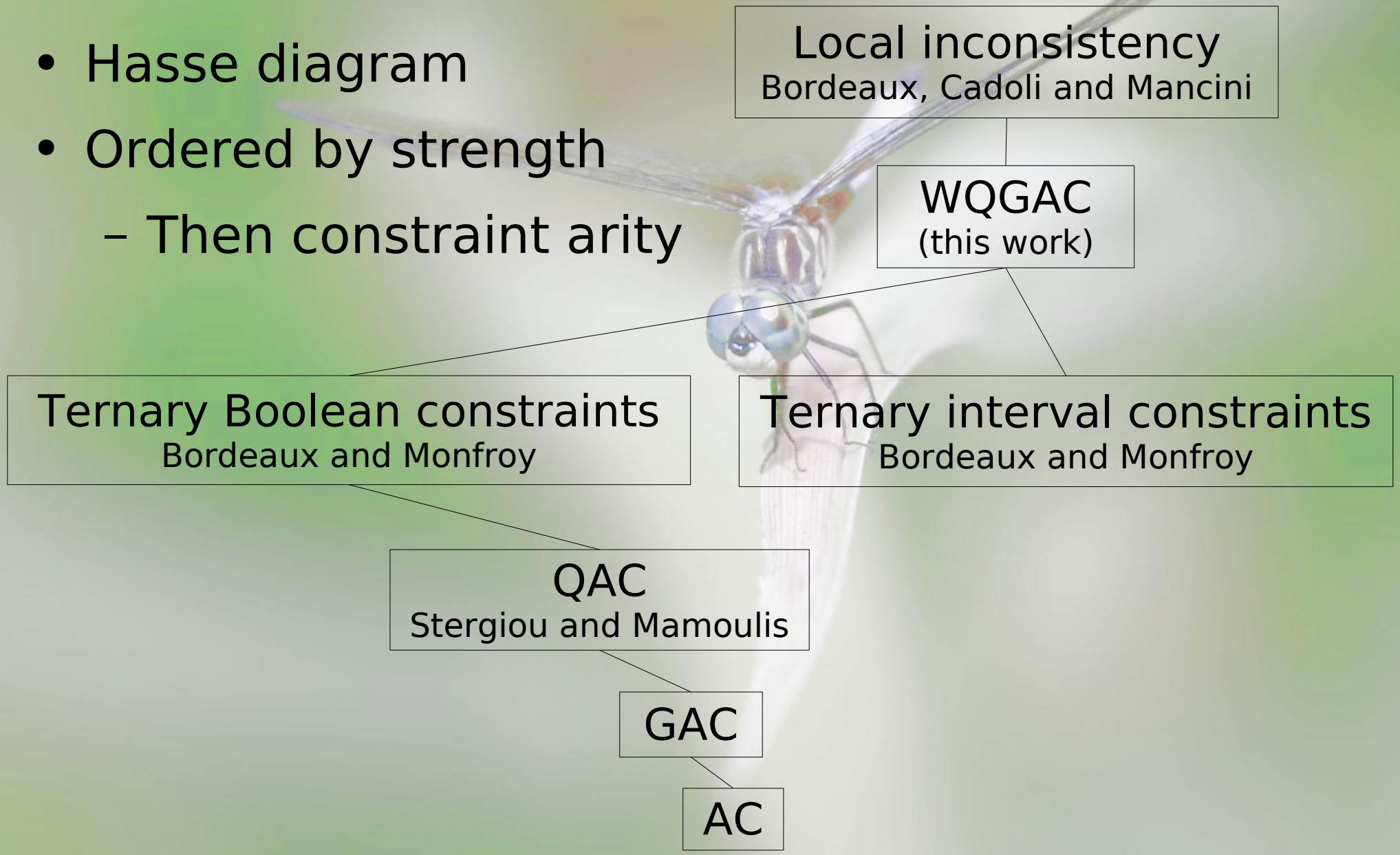


Consistency notions

- Hasse diagram
- Ordered by strength
  - Then constraint arity



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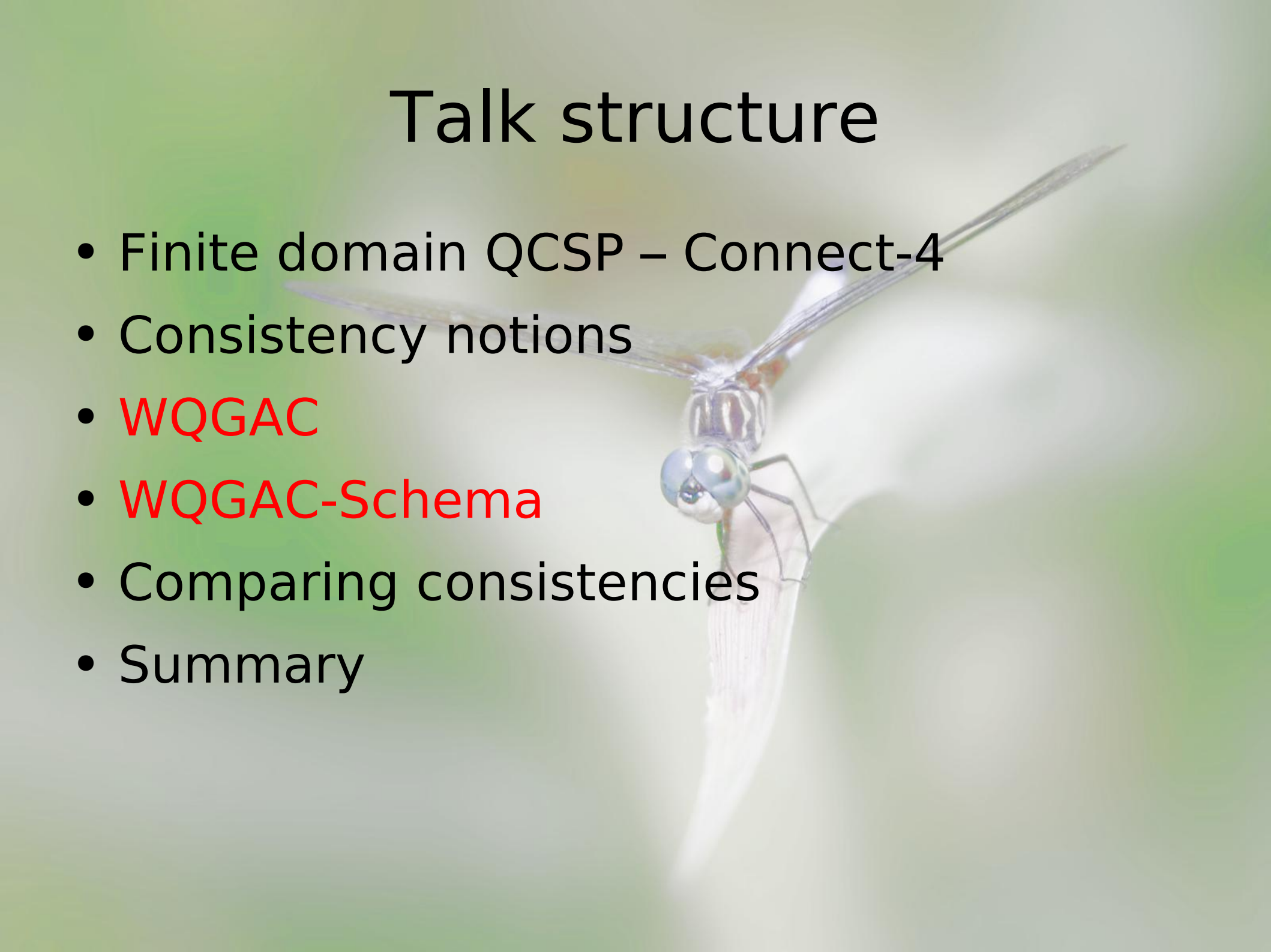

WQGAC

- With GAC each value has a supporting tuple
- With WQGAC each value has a supporting tuple for each combination of values of inner universals

$\exists a \forall b \exists c : a \Leftrightarrow b \wedge c$

Supporting a=0:

a	b	c
0	0	0
0	0	1
0	1	0
1	1	1

WQGAC

- With GAC each value has a supporting tuple
- With WQGAC each value has a supporting tuple for each combination of values of inner universals

$\exists a \forall b \exists c : a \Leftrightarrow b \wedge c$

Supporting a=1:

a	b	c
0	0	0
0	0	1
0	1	0
1	1	1

WQGAC-Schema

- Based on GAC-Schema (Bessière and Régin)
- Time: $O(n^2d^n)$
- Space: $O(n^2d^{u+1})$
- Generalization of GAC-Schema
- Multidirectional

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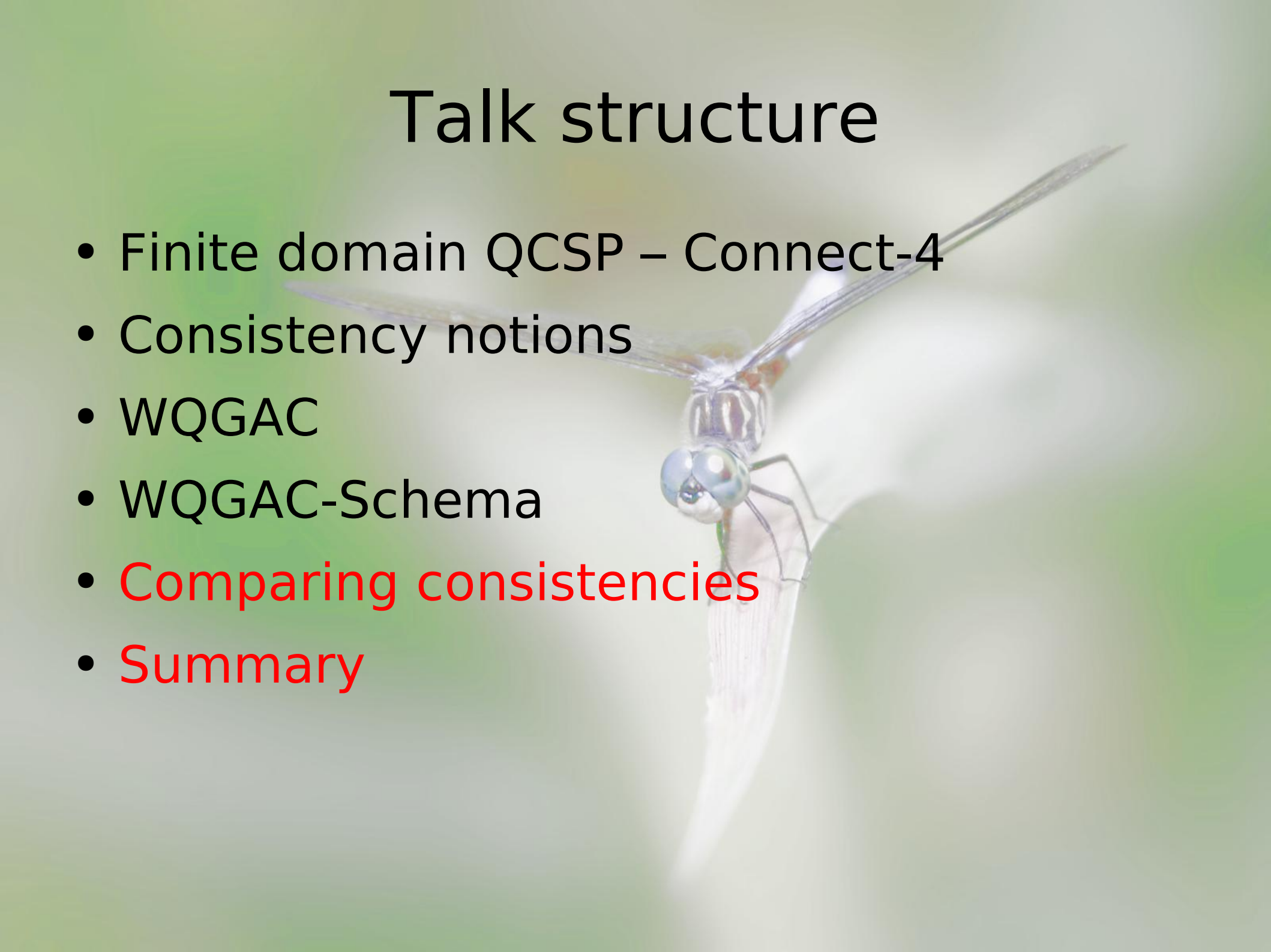

Comparing consistencies

Consistency	Inference	Resources used
QAC on the hidden variable encoding	none	
GAC	none	
WQGAC	1,3,5..7 pruned from *grey1*	0.046s, checked 15.2% of all 7^5 tuples.
B,C & M inconsistency	1,3,5..7 pruned from *grey1* 1,3,6,7 pruned from *grey2* 1,3,7 pruned from *grey3*	

Comparing consistencies

- WQGAC weak
  - For each value, set of supporting tuples
  - May not be part of one strategy

$\forall a \exists b \forall c \in \{0,1\}$

a=0 supported by:

Value of b is different

a	b	c
0	0	0
0	1	1
1	0	1
1	1	0

Summary

- Reasonably powerful algorithm for local reasoning in finite domain QCSP
- Future work
  - Tuple/tree mismatch
  - Different support structure

Thank you

- Any questions?