Quick Review over the Last Lecture

Wave / particle duality of an electron:

<table>
<thead>
<tr>
<th></th>
<th>Particle nature</th>
<th>Wave nature</th>
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</thead>
<tbody>
<tr>
<td>Kinetic energy</td>
<td></td>
<td></td>
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<tr>
<td>Momentum</td>
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</tr>
</tbody>
</table>

Brillouin zone (1st & 2nd):

Fermi-Dirac distribution ($T$-dependence):

\[
f(E) = \mu \left(1 + \frac{1}{e^{\frac{E-E_0}{kT}} + 1}\right)^{-1}
\]
Contents of Introductory Nanotechnology

First half of the course:
Basic condensed matter physics
1. Why solids are solid?
2. What is the most common atom on the earth?
3. How does an electron travel in a material?
4. How does lattices vibrate thermally?
5. What is a semi-conductor?
6. How does an electron tunnel through a barrier?
7. Why does a magnet attract / retract?
8. What happens at interfaces?

Second half of the course:
Introduction to nanotechnology (nano-fabrication / application)

How Does an Electron Travel in a Material?

- Group / phase velocity
  - Effective mass
    - Hall effect
  - Harmonic oscillator
- Longitudinal / transverse waves
  - Acoustic / optical modes
  - Photon / phonon
How Fast a Free Electron Can Travel?

**Electron wave under a uniform \( \mathbf{E} \):**

Phase-travel speed in an electron wave:

Group velocity:

\[
v_g = \frac{d\omega}{dk}
\]

Here, energy of an electron wave is

\[
E = h\nu = \hbar \omega
\]

Accordingly,

\[
v_g = \frac{1}{\hbar} \frac{dE}{dk}
\]

Therefore, electron wave velocity depends on gradient of energy curve \( E(k) \).

**Equation of Motion for an Electron with \( k \):**

For an electron wave travelling along \( \mathbf{E} \):

\[
\frac{dv_g}{dt} = \frac{1}{\hbar} \frac{d}{dt} \left( \frac{dE}{dk} \right) = \frac{1}{\hbar} \frac{d}{dk} \left( \frac{dE}{dk} \right) \frac{dt}{dt} = \frac{1}{\hbar} \frac{d^2E}{dk^2} \frac{dt}{dt}
\]

Under \( \mathbf{E} \), an electron is accelerated by a force of \(-q\mathbf{E}\).

In \( \Delta t \), an electron travels \( v_g \Delta t \), and hence \( \mathbf{E} \) applies work of \((-q\mathbf{E})(v_g \Delta t)\).

Therefore, energy increase \( \Delta E \) is written by

\[
\Delta E = -qE v_g \Delta t = -qE \frac{1}{\hbar} \frac{dE}{dk} \Delta t
\]

At the same time \( \Delta E \) is defined to be

\[
\Delta E = \frac{dE}{dk} \Delta k
\]

From these equations,

\[
\Delta k = -\frac{1}{\hbar} qE \Delta t \quad \therefore \quad \frac{dk}{dt} = -\frac{1}{\hbar} qE \quad \therefore \quad \frac{dk}{dt} = -qE
\]

→ Equation of motion for an electron with \( k \).
**Effective Mass**

By substituting $\hbar \frac{dk}{dt} = -qE$ into $\frac{d\mathbf{v}_g}{dt} = \frac{1}{\hbar} \frac{d^2E}{dk^2} \frac{dk}{dt}$

$$\frac{d\mathbf{v}_g}{dt} = -\frac{1}{\hbar^2} \frac{d^2E}{dk^2} qE$$

By comparing with acceleration for a free electron :

$$\frac{dv}{dt} = -\frac{1}{m} qE$$

$$m^* = \frac{\hbar^2}{\left( \frac{d^2E}{dk^2} \right)}$$

→ Effective mass

---

**Hall Effect**

Under application of both an electrical current $i$ and magnetic field $B$:

$$\mathbf{F} = -qE + v \times B$$

$\mathbf{E} = \frac{1}{qn} i B_z \frac{l}{l}$

$\mathbf{V}_{Hy} = \frac{1}{qn} i B_z \frac{l}{l}$

Hall coefficient
Harmonic Oscillator

Lattice vibration in a crystal:

Hooke's law:

\[ M \frac{d^2 u}{dt^2} = -kx \]

Here, we define

\[ \omega = \sqrt{\frac{k}{M}} \quad \therefore \frac{d^2 u}{dt^2} = -\omega^2 u \]

\[ : u(t) = A \sin(\omega t + \alpha) \]

→ 1D harmonic oscillation

Strain

Displacement per unit length:

\[ \delta = \left( \frac{\partial u}{\partial x} \right) dx = \frac{\partial u}{\partial x} \]

Young's law (stress = Young's modulus × strain):

\[ \frac{F}{S} = E_Y \frac{\partial u}{\partial x} \quad S : \text{area} \]

Here,

\[ +F(x + dx) = F(x) + \left( \frac{\partial F}{\partial x} \right) dx + \cdots \]

For density of \( \rho_i \)

\[ \rho S dx \frac{\partial^2 u}{\partial t^2} = \frac{\partial F}{\partial x} dx \]

\[ \therefore \frac{\partial^2 u}{\partial t^2} = \frac{E_Y}{\rho} \frac{\partial^2 u}{\partial x^2} = \nu_i \frac{\partial^2 u}{\partial x^2} \]

→ Wave equation in an elastomer

Therefore, velocity of a strain wave (acoustic velocity):

\[ v_i = \sqrt{\frac{E_Y}{\rho}} \]
Longitudinal / Transverse Waves

Longitudinal wave: vibrations along or parallel to their direction of travel

Transverse wave: vibrations perpendicular to their direction of travel

Transverse Wave

* http://www12.plala.or.jp/ksp/wave/waves/
Longitudinal Wave

Propagation direction
Amplitude

Sparse
Dense
Sparse
Sparse
Dense
Sparse

Acoustic / Optical Modes

For a crystal consisting of 2 elements (k : spring constant between atoms):

\[
\begin{align*}
M \frac{d^2 u_n}{dt^2} &= k\{(v_n - u_n) + (v_{n-1} - u_n)\} \\
m \frac{d^2 v_n}{dt^2} &= k\{(u_{n+1} - v_n) + (u_n - v_n)\}
\end{align*}
\]

By assuming,

\[
\begin{align*}
\begin{cases}
  u_n(\text{na}, t) = A \exp\{i(\omega t - qa)\} \\
v_n(\text{na}, t) = B \exp\{i(\omega t - qa)\}
\end{cases}
\end{align*}
\]

\[
\begin{align*}
-\omega^2 A &= kB\{1 + \exp(iqa)\} - 2kB \\
-\omega^2 B &= kA\{\exp(-iqa) + 1\} - 2kB
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
\left(2k - \omega^2\right)A - k\{1 + \exp(iqa)\}B = 0 \\
-k\{\exp(-iqa) + 1\} A + \left(2k - \omega^2\right)B = 0
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\begin{vmatrix}
2k - \omega^2 & -k\{1 + \exp(iqa)\} \\
-k\{\exp(-iqa) + 1\} & 2k - \omega^2
\end{vmatrix} = 0
\end{align*}
\]
Therefore,  \( \omega^2 = k \left( \frac{1}{M} + \frac{1}{m} \right) \pm k \sqrt{\left( \frac{1}{M} + \frac{1}{m} \right)^2 - \frac{4}{Mm} \sin^2 qa} \)

For \( qa = 0 \),
\[
\begin{align*}
\omega_+ &= \sqrt{2k \left( \frac{1}{M} + \frac{1}{m} \right)} \\
\omega_- &= 0
\end{align*}
\]

For \( qa \sim 0 \),
\[
\begin{align*}
\omega_+ &\approx \sqrt{\frac{2k}{M} \left( \frac{1}{M} + \frac{1}{m} \right)} \\
\omega_- &\approx \frac{k/2}{M + m} qa
\end{align*}
\]

For \( qa = \pi \),
\[
\begin{align*}
\omega_+ &= \frac{2k}{m} \\
\omega_- &= \frac{2k}{M}
\end{align*}
\]


### Why Acoustic / Optical Modes ?

Oscillation amplitude ratio between \( M \) and \( m \) (\( A / B \)):

**Optical mode**: \( \frac{m}{M} \)

Neighbouring atoms changes their position in opposite directions, of which amplitude is larger for \( m \) and smaller for \( M \), however, the centre of gravity stays in the same position.

**Acoustic mode**: 1

All the atoms move in parallel.
Photon / Phonon

Quantum hypothesis by M. Planck (black-body radiation):
\[ E = \frac{1}{2} \hbar \nu + n\hbar \nu \quad (n = 0,1,2,...) \]

Here, \( \hbar \nu \) : energy quantum (photon)
mass : 0, spin : 1

Similarly, for an elastic wave, quasi-particle (phonon) has been introduced by P. J. W. Debye.
\[ E = \frac{1}{2} \hbar \omega + n\hbar \omega \quad (n = 0,1,2,...) \]

Oscillation amplitude : larger \( \rightarrow \) number of phonons : larger

What is a conductor?

Number of electron states (including spins) in the 1st Brillouin zone:
\[ \int_{-\pi/a}^{\pi/a} \frac{L}{2\pi} dk = \frac{2L}{a} = 2N \quad \left( N = \frac{L}{a} \right) \]

Here, \( N \) : Number of atoms for a monovalent metal
As there are \( N \) electrons, they fill half of the states.

By applying an electrical field \( E \), the occupied states become asymmetric.
- \( E \) increases asymmetry.
- Elastic scattering with phonon / non-elastic scattering decreases asymmetry.
  \( \rightarrow \) Stable asymmetry
  \( \rightarrow \) Constant current flow
  \( \rightarrow \) Conductor :
    Only bottom of the band is filled by electrons with unoccupied upper band.