Fixed boundary codes

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Nonlinear Partial Differential Equation

$$\Delta^* = R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right) + \frac{\partial^2\psi}{\partial z^2}$$

$$F(R,z) = -\mu_0 R^2 p' - ff'$$

f, p fn of ψ

p', ff': inputs

picard iteration

Choose $\psi^0(R,z)$

$\downarrow$

$\rightarrow F(\psi^n)$

$\downarrow$

Solve $\psi^{n+1} = (\Delta^*)^{-1}(F(\psi^n))$

$\downarrow$ *

Check $|\psi^{n+1} - \psi^n|_{max}$ (if not, return to $F(\psi^n)$)

$\downarrow$ $< \varepsilon$ small tolerance

finished

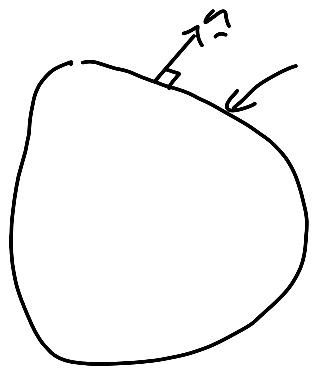
See also Newton

Oscillating solutions: $\psi^{n+1} \rightarrow \alpha\psi^{n+1} + (1-\alpha)\psi^n$

blending $0 < \alpha \leq 1$

Boundary condition

fixed



$$\underset{\sim}{B} \cdot \underset{\sim}{\hat{n}} = 0$$

$$\underset{\sim}{B} = \nabla\psi \times \nabla\alpha$$

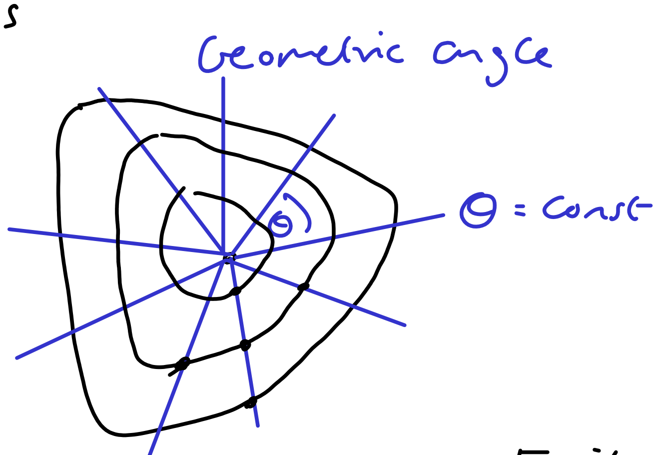
$$(\nabla\psi \times \nabla\alpha) \cdot \underset{\sim}{n} = 0$$

$$(\underset{\sim}{n} \times \nabla\psi) \cdot \nabla\alpha = 0$$

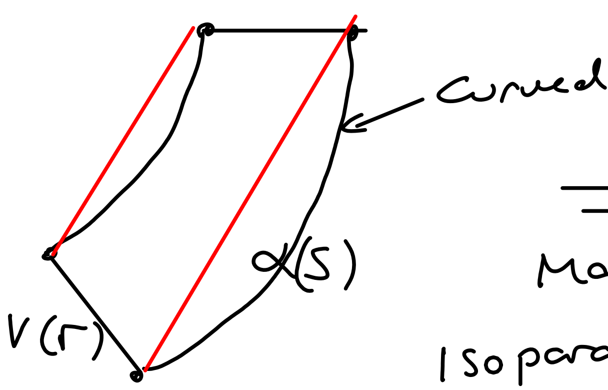
$$\underset{\sim}{n} \propto \nabla\psi$$

ψ = const on surface

Coordinates



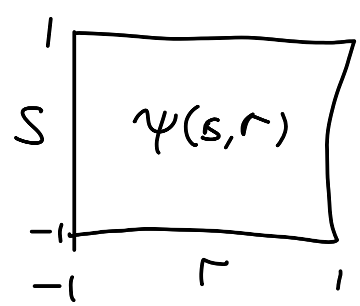
Finite Element



$\Longrightarrow$

Mapping

Isoparametric



$$R = V(r)\,\alpha(S)\cos(\pi S)$$

$$Z = V(r)\,\alpha(S)\sin(\pi S)$$

shape

$0 \leq V \leq 1$

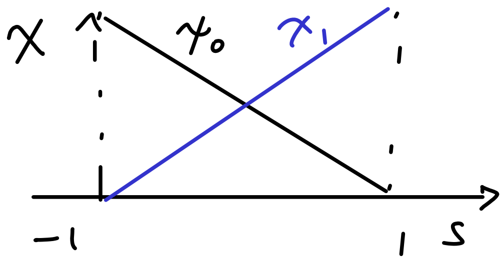
axis edge

$$\psi(s,r) = \sum_i \psi_i \, \chi(s,r)$$ e.g. Hermite polynomials

Insert into G-S equation

$$\int_V \chi \, \Delta^*(\psi) \, dV = \int_V \chi \, F(R,\psi) \, dV$$

weak form



$$\underline{\underline{K}}(\psi_i) = \underline{b}$$ linear matrix inversion

→ standard methods