Free boundary solvers

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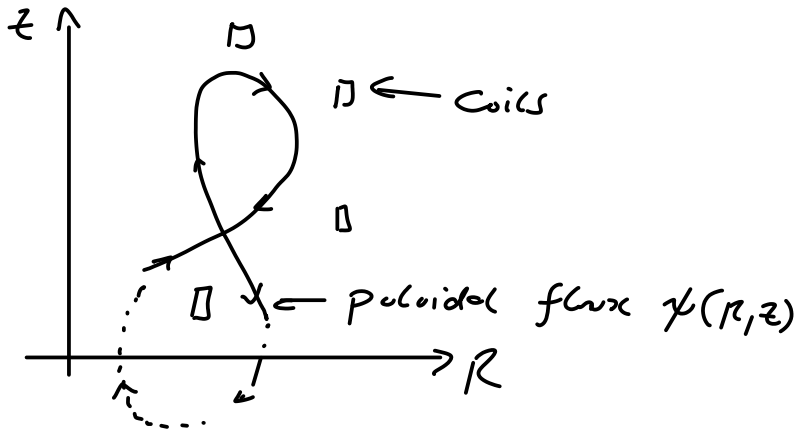


$$\psi = \psi_{plasma} + \psi_{coils}$$

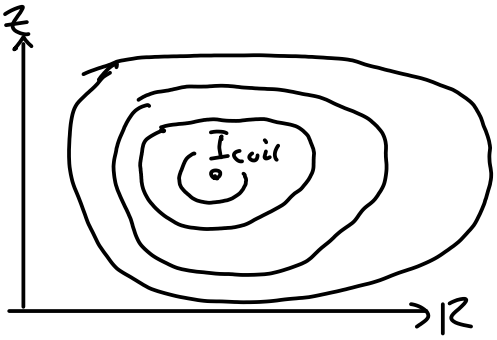
$$\Delta^* \psi_{plasma} = -\mu_0 R J_\phi(R,z)$$

$$\psi_{coils} = \sum_{coil} G(R,z;R_{coil},z_{coil})\, I_{coil}$$

$$J_{coil} = I_{coil}\, \delta(R-R_{coil})\, \delta(z-z_{coil})$$

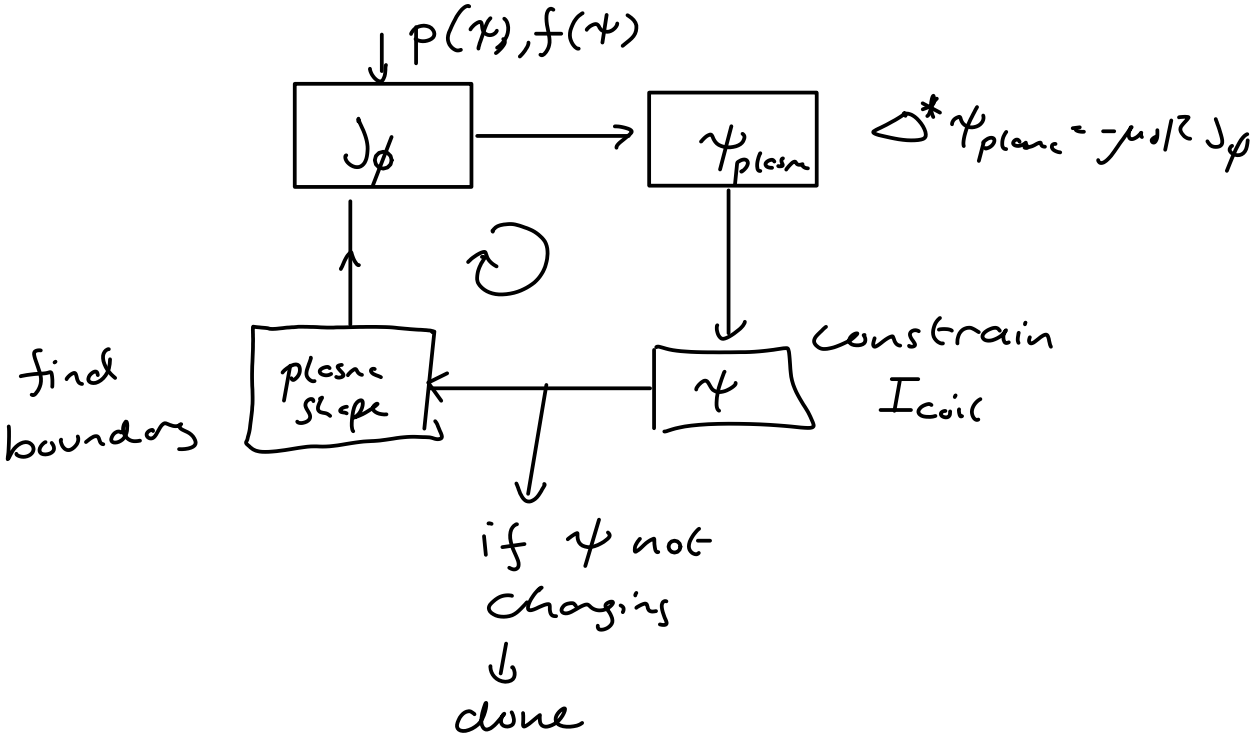
$$G(R,z;R_{coil},z_{coil}) = -\frac{1}{2\pi}\sqrt{R R_{coil}}\,\frac{1}{k}\left[\left(2-k^2\right)K(k) - 2E(k)\right]$$

(K: elliptic integral of the first kind; E: elliptic integral of the second kind)

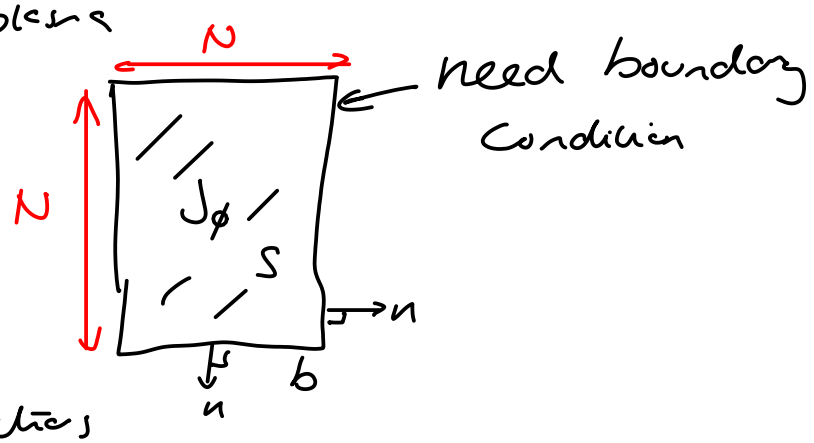


$$k = \frac{4 R R_{coil}}{(R+R_{coil})^2 + (z-z_{coil})^2}$$

Free boundary solver loop



1) Calculating ψ_{plasma}



a) Greens function

$$\psi(R_b, z_b) = \int_S G(R_b, z_b; R, z)\, J_\phi(R,z)\, dR\, dz$$

$\sim N^2$

$\Rightarrow$ total work $\sim N^3$ $\rightarrow$ very slow

b) Von Hagenow's method

$$\Delta^* u = -\mu_0 R J_\phi \qquad u = 0 \quad \text{on boundary}$$

$$\int_S G(R_b, z_b; R, z)\, J_\phi \, dR\, dz = \oint_b \frac{G(R_b, z_b; R_b', z_b')}{R} \frac{\partial u}{\partial n} \, dl$$

$$\rightarrow \text{cost} \sim N^2$$

2) Coil current, control

Constraints

- Isoflux $\quad \psi(R_1, z_1) = \psi(R_2, z_2)$

- X-point location

$$B_R(R_x, R_z) = 0 \qquad B_z(R_x, z_x) = 0$$

Solve a least squares problem

$$\varepsilon = \int \sum_{coil} \Big(G(R_1, z_1; R_{coil}, z_{coil}) - G(R_2, z_2; R_{coil}, z_{coil}) \Delta I_{coil} - \Delta\psi \Big)^2$$

(Isoflux; $\psi(R_1, z_1) - \psi(R_2, z_2)$)

$$+ \int \sum_{coil} G_{B_r}(R_x, z_x; R_{coil}, z_{coil}) \Delta I_{coil} - B_R(R_x, z_x) \Big]^2$$

$$+ \int \sum_{coil} G_{B_z}(R_x, z_x; R_{coil}, z_{coil}) \Delta I_{coil} - B_z(R_x, z_x) \Big]^2$$

(X-point)

$$\text{Regularisation} + \gamma^2 \sum_{coil} \Delta I_{coil}^2$$

$$G_{B_r} = -\frac{1}{R} \frac{\partial G}{\partial z}$$

$$G_{B_z} = \frac{1}{R} \frac{\partial G}{\partial R}$$

3) find plasma boundary, axis

- Current only inside plasma
- Specify profiles in normalised ψ

$$\psi_n = \frac{\psi - \psi_{axis}}{\psi_{boundary} - \psi_{axis}}$$