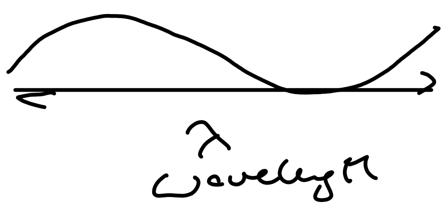
Plasma waves

Contents

- Waves in homogenous plasma
- Shear Alfven waves
- Fast and slow magnetosonic waves

Stationary $\underset{\sim}{u}_0 = 0$ uniform (homogenous)

$\nabla p_0 = 0 \quad \nabla \rho_0 = 0$



$k = \frac{2\pi}{\lambda}$ $\omega = 2\pi f$

frequency

$$\rho = \rho_0 + \delta\rho(x, t)$$

$$\delta\rho(\underset{\sim}{x}, t) = \iint \delta\hat{\rho}(\underset{\sim}{k}, \omega)\, e^{i(\underset{\sim}{k}\cdot\underset{\sim}{x} - \omega t)}\, d\underset{\sim}{k}\, d\omega$$

Fourier transform

$$\frac{\partial}{\partial t}\delta\rho = -\underset{\sim}{u}_0 \cdot \nabla\delta\rho - \rho_0 \nabla\cdot\delta\underset{\sim}{u} - \delta\underset{\sim}{u}\cdot\nabla\rho_0 - \delta\rho\,\nabla\cdot\underset{\sim}{u}_0$$

$$\frac{\partial}{\partial t}\iint \delta\hat{\rho}(\underset{\sim}{k}, \omega)\, e^{i(\underset{\sim}{k}\cdot\underset{\sim}{x} - \omega t)}\, d\underset{\sim}{k}\, d\omega = -\rho_0 \nabla\cdot \iint \delta\hat{\underset{\sim}{u}}(\underset{\sim}{k}, \omega)\, e^{i(kx - \omega t)}\, dk\, d\omega$$

$$\nabla\cdot \to i\underset{\sim}{k}\cdot$$

$$\frac{\partial}{\partial t} \to -i\omega$$

$$\iint \underbrace{\left[-i\omega\delta\hat{\rho} + \rho_0 i\underset{\sim}{k}\cdot\delta\hat{\underset{\sim}{u}}\right]}_{=0} e^{i(\underset{\sim}{k}\cdot\underset{\sim}{x} - \omega t)}\, d\underset{\sim}{k}\, d\omega = 0$$

$$-i\omega\,\delta\hat{\rho} = -\rho_0\, i\underset{\sim}{k}\cdot\delta\underline{u}$$

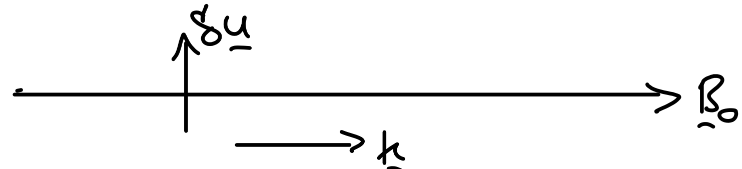
$*$ $$-i\omega\,\delta\hat{\underline{u}} = -\frac{\delta\hat{p}}{\rho_0}\,i\underset{\sim}{k} + \frac{1}{\mu_0\rho_0}\left(i\underset{\sim}{k}\times\delta\underset{\sim}{B}\right)\times\underset{\sim}{B}_0$$

$$-i\omega\,\delta\hat{p} = -\gamma P_0\, i\underset{\sim}{k}\cdot\delta\hat{\underline{u}}$$

$$-i\omega\,\delta\hat{B} = i\underset{\sim}{k}\times\left(\delta\hat{\underline{u}}\times\underset{\sim}{B}_0\right)$$

$$-\omega^2\delta\underline{u} = \underbrace{\frac{-\gamma P_0}{\rho_0}}_{\text{sound speed } c_s^2}(\underset{\sim}{k}\cdot\delta\underline{u})\underset{\sim}{k} - \underbrace{\frac{B_0^2}{\mu_0\rho_0}}_{\text{Alfven speed } V_A^2}(\underset{\sim}{k}\cdot\delta\underline{u})\underset{\sim}{k}$$
$$+\frac{1}{\mu_0\rho_0}\left[-(\underset{\sim}{B}_0\cdot\underset{\sim}{k})^2\delta\underline{u} + (\underset{\sim}{B}_0\cdot\delta\underline{u})(\underset{\sim}{k}\cdot\underset{\sim}{B}_0)\underset{\sim}{k} + (\underset{\sim}{B}_0\cdot\underset{\sim}{k})(\underset{\sim}{k}\cdot\delta\underline{u})\underset{\sim}{B}_0\right]$$

$\underset{\sim}{k}$ along $\underset{\sim}{B}$ $\quad$ $\underset{\sim}{k}\cdot\delta\underline{u} = 0$ $\quad$ $\underset{\sim}{k}\cdot\underset{\sim}{B}\neq 0$ $\quad$ $\delta\underline{u}\cdot\underset{\sim}{B} = 0$



$$-\omega^2\delta\underline{u} = -\frac{B_0^2}{\mu_0\rho_0}\,\delta u\, k^2$$

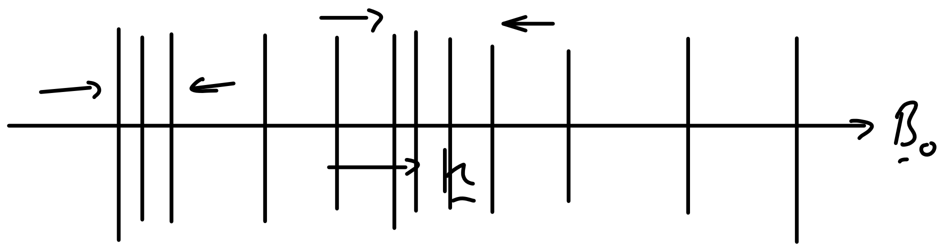
$$\frac{\omega^2}{k^2} = \frac{B_0^2}{\mu_0\rho_0} = V_A^2 \quad \text{Alfven speed}$$

Shear Alfven wave

$\underset{\sim}{k}$ along $\underset{\sim}{B}$ $\qquad$ $\delta\underline{u}$ along $\underset{\sim}{B}$

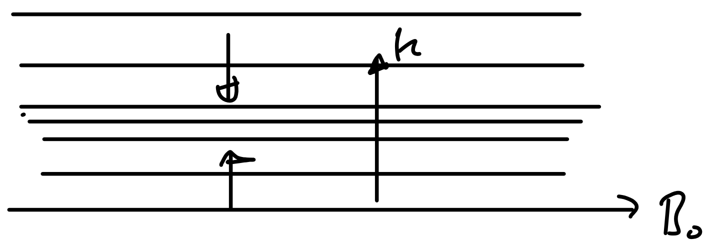
$$\frac{\omega^2}{k^2} = c_s^2$$

$$-\omega^2\delta u = -c_s^2k^2\delta u - V_A^2k^2\delta u$$
$$+\frac{1}{\mu_0\rho_0}\left[-B_0^2k^2\delta u + B_0^2k^2\delta u + B_0^2k^2\delta u\right]$$

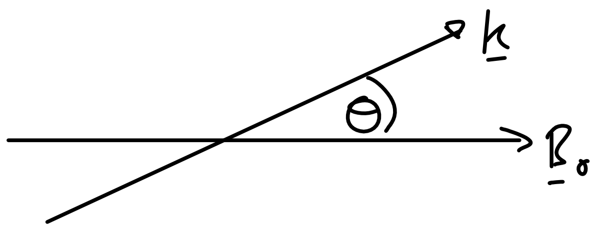


$\underset{\sim}{k} \perp \underset{\sim}{B_0}$ $\quad \underset{\sim}{k} \cdot \underset{\sim}{B} = 0$ $\quad \delta \underset{\sim}{u}$ along k $\quad \delta \underset{\sim}{u} \cdot \underset{\sim}{B} = 0$

$$\frac{\omega^2}{k^2} = C_s^2 + V_A^2$$ fast wave



General case



Slow: $$\frac{\omega^2}{k^2} = \frac{1}{2}\left[V_A^2 + C_s^2 - \sqrt{(V_A^2 + C_s^2) - 4 V_A^2 C_s^2 \cos^2\theta} \right]$$

$\longrightarrow$ sound wave if $\theta \to 0$

Fast: $$\frac{\omega^2}{k^2} = \frac{1}{2} \left| V_A^2 + C_s^2 + \sqrt{(V_A^2 + C_s^2) - 4 V_A^2 C_s^2 \cos\theta} \right.$$

$\longrightarrow$ fast wave

Magnetosonic