The Energy Principle

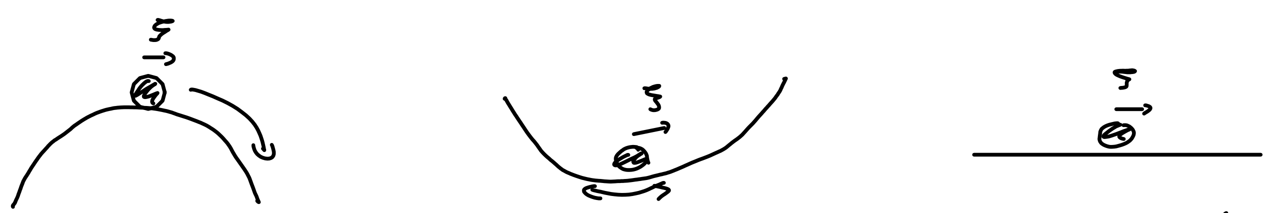
Contents

- Testing for plasma stability
- Intuitive form of energy principle
- Good and bad curvature

Start with equilibrium

Give small perturbation (displacement ξ)

Energy is conserved $\Rightarrow$ Potential energy lost is kinetic energy gained



Unstable	Stable	Marginal
$\omega^2 < 0$	$\omega^2 > 0$	$\omega^2 = 0$

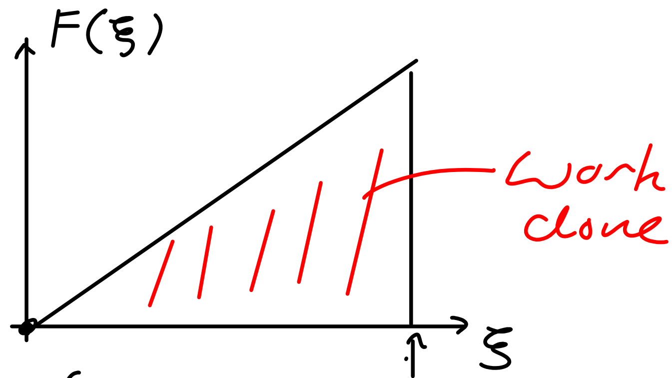
potential energy of the plasma change due to small perturbation $\underline{\xi}$

- if any ξ can be found which causes potential energy $\delta W < 0 \Rightarrow$ <u>unstable</u>
- A stable plasma must have $\delta W > 0$ for all $\underline{\xi}$

Normal mode formulation

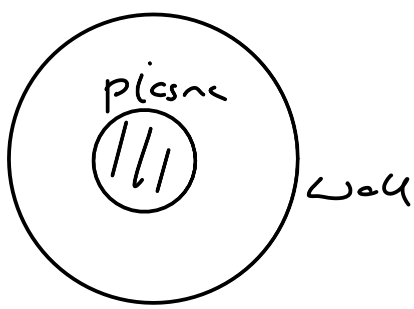
$$-\rho_0 \omega^2 \underline{\xi} = F(\underline{\xi})$$

Force × distance → work done



$$\delta W = -\frac{1}{2}\int \underline{\xi}\cdot F(\underline{\xi})\, d\underline{x}$$

plasma + vacuum



$\underline{\xi} = 0$

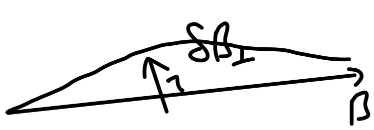
Extended Energy principle

$$\delta W = \delta W_p + \delta W_v + \delta W_s$$

plasma vacuum surface

$$\delta W_p = \frac{1}{2}\int d\underline{x} \Big[\frac{|\delta B_\perp|^2}{\mu_0} \geq 0$$

Shear Alfvén energy in perturbed B field
field-line bending



$\delta B_\perp = [\underline{b} \times \delta \underline{B}] \times \underline{b}$

$$+ \frac{B_0^2}{\mu_0}|\nabla\cdot\underline{\xi}_\perp + \underline{\xi}_\perp\cdot\underline{\kappa}|^2 \geq 0$$

Magnetic Compression

$$+ \gamma P_0 |\nabla\cdot\underline{\xi}|^2 \geq 0$$

fluid compression

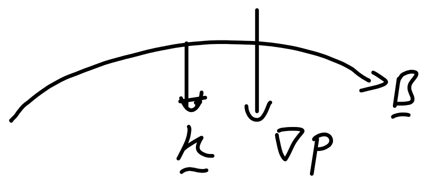
pressure (ballooning) drive $-2(\underline{\xi}_\perp\cdot\nabla P_0)(\underline{\kappa}\cdot\underline{\xi}_\perp^*)$ > 0 or < 0

$$- \delta \underline{B} \cdot (\underline{\xi}_\perp \times \underline{b}) \underline{\underline{J_\parallel}} \Big] > 0 \text{ or } < 0$$

kink drive

Good vs bad curvature

$$-2(\underline{\xi}_\perp \cdot \nabla p_0)(\underline{\kappa} \cdot \underline{\xi}_\perp^*) < 0 \quad \text{if} \quad \nabla p \cdot \underline{\kappa} > 0$$



unstable

bad curvature

Vacuum contribution

$$\delta W_V = \frac{1}{2} \int dx \frac{|\delta \underline{B}|^2}{\mu_0}$$

Vacuum magnetic field energy

Surface contribution

$$\delta W_S = -\frac{1}{2} \oint \left[\gamma p \nabla \cdot \underline{\xi} - \frac{\underline{B}_0 \cdot \delta \underline{B}}{\mu_0} \right] \underline{\xi}^* \cdot d\underline{s}$$

$\sim$ surface current