Ballooning modes

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- Field line bending
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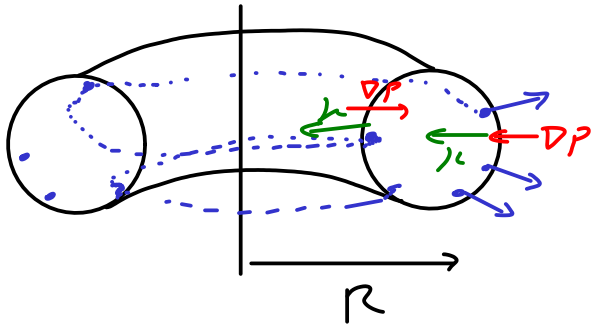
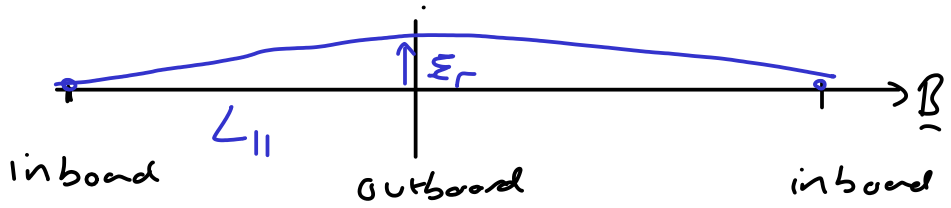


ballooning on bad curvature ("outboard") side



$$L_\parallel \sim \frac{1}{\epsilon} q\, 2\pi R$$

$$\frac{\delta B_\perp}{B} \sim \frac{\xi_r}{L_\parallel}$$

Energy principle

$$\delta W = \frac{1}{2}\int d\underline{x}\left[\frac{|\delta \underline{B}_\perp|^2}{\mu_0} - 2\underbrace{(\nabla p\cdot \underline{\xi}_\perp)}_{\frac{dp}{dr}\cdot\xi_r}\underbrace{(\underline{\underline{\kappa}}\cdot \underline{\xi}_\perp^*)}_{\frac{-1}{R}\xi_r}\right]$$

$$\delta W \simeq \frac{1}{2}\int dx\left[\frac{|B_0\,\xi_r/L_\parallel|^2}{\mu_0} + 2\frac{dp}{dr}\frac{\xi_r^2}{R}\right]$$

Unstable if $-2\frac{dP}{dr}\frac{\xi_r^2}{R} > \frac{|B_0 \xi_r / L_{11}|^2}{\mu_0}$ $\quad L_{11} = q\pi R$

$$-\frac{dP}{dr} > \underbrace{\frac{B_0^2}{2\mu_0}}\ \frac{1}{\pi^2 q^2 R}$$

Average pressure gradient

$$-\frac{dP}{dr} \sim \frac{P_0}{a} \qquad \frac{P_0}{(B^2/2\mu_0)} > \frac{a}{\pi^2 q^2 R}$$

unstable

$$\beta = \frac{2\mu_0 P}{B_0^2} < \frac{a}{\pi^2 q^2 R}$$

e.g. $q=2$ $\quad \frac{a}{R} \sim 1/3$

$\Rightarrow \beta \sim 1\%$