Flux surface perturbations

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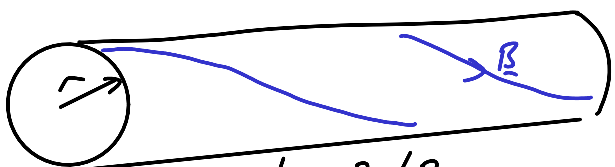
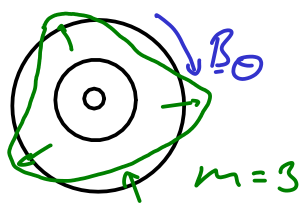
$R >> r \qquad \varepsilon = \frac{r}{R} << 1$

inverse aspect ratio



$\phi = z/R_0$



$$B_r = -\frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

$$B_\theta = \frac{\partial \psi}{\partial r}$$

Small change in poloidal flux, $\delta\psi$

$$\delta B_r = -\frac{1}{r}\frac{\partial \delta\psi}{\partial \theta}$$

$$\delta B_\theta = \frac{\partial \delta\psi}{\partial r}$$

$$J_\phi = \frac{1}{\mu_0}(\nabla \times B_\theta)_\phi$$

$$\delta J_\phi = \frac{1}{\mu_0}\nabla^2 \delta\psi$$ toroidal current density

$$\delta\psi(r,\theta,\phi) = \delta\hat{\psi}(r)\, e^{i(m\theta - n\phi)}$$

Slow instability $\Rightarrow$ $\underline{J} \times \underline{B} = \nabla p$

Take curl
$$\nabla \times (\underline{J} \times \underline{B}) = 0$$

$$\nabla \times (\delta \underline{J} \times \underline{B}_0 + \underline{J}_0 \times \delta \underline{B}) = 0$$

Small

$$\delta \underline{J}\,\cancel{(\nabla\cdot\underline{B}_0)} - \underline{B}_0\cancel{(\nabla\cdot\delta\underline{J})} + (\underline{B}_0\cdot\nabla)\delta\underline{J} - (\delta\underline{J}\cdot\nabla)\underline{B}_0$$
$$+\ \underline{J}_0\cancel{(\nabla\cdot\underline{B}_0)} - \delta\underline{B}\cancel{(\nabla\cdot\underline{J}_0)} + (\delta\underline{B}\cdot\nabla)\underline{J}_0 - (\underline{J}_0\cdot\nabla)\delta\underline{B} = 0$$

$$(\underline{B}_0\cdot\nabla)\delta\underline{J} + (\delta\underline{B}\cdot\nabla)\underline{J}_0 - (\underline{J}_0\cdot\nabla)\delta\underline{B} = 0$$

Take ϕ component

$$\underline{(B_0\cdot\nabla)\,\delta J_\phi} + \underline{(\delta\underline{B}\cdot\nabla)\,J_\phi} \simeq 0$$

Large aspect ratio $\frac{R_0}{r} \gg 1$ $\quad J_\phi \sim J_\phi(r)$

$$\delta B_r \frac{dJ_\phi}{dr} = -\frac{1}{r}\underbrace{\frac{\partial \delta\psi}{\partial\theta}}_{im\,\delta\psi}\frac{dJ_\phi}{dr}$$

$$B_0 = B_\theta \hat{\underline{\theta}} + B_\phi \hat{\phi}$$

$$\nabla = \frac{\partial}{\partial r}\hat{\underline{r}} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\underline{\theta}} + \frac{1}{R_0}\frac{\partial}{\partial\phi}\hat{\phi}$$

$$\Rightarrow (B_0\cdot\nabla)\,\delta J_\phi = \left[B_\theta \frac{1}{r}\frac{\partial}{\partial\theta} + B_\phi \frac{1}{R_0}\frac{\partial}{\partial\phi}\right]\delta J_\phi$$

$$= \left[B_\theta \frac{im}{r} - B_\phi \frac{in}{R_0}\right]\delta J_\phi$$

$$\left[B_\theta \frac{im}{r} - \underbrace{B_\phi \frac{in}{R_0}} \right] \delta J_\phi = \frac{im}{r} \delta\psi \frac{dJ_\phi}{dr}$$

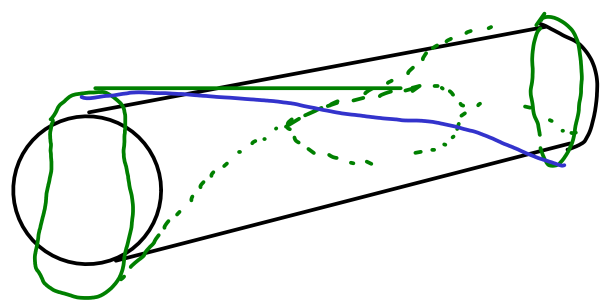
$$q = \frac{B_\phi r}{B_\theta R_0}$$ safety factor

$$m \frac{B_\theta}{\cancel{r}} \left[1 - q\frac{n}{m} \right] \underbrace{\delta J_\phi}_{\frac{1}{\mu_0}\nabla^2 \delta\psi} = \frac{m}{\cancel{r}} \delta\psi \frac{dJ_\phi}{dr}$$

$$\boxed{\delta J_\phi = \frac{\delta\psi \frac{dJ_\phi}{dr}}{B_\theta \left[1 - q\, n/m \right]}}$$

Cylindrical Tearing Mode Equation

$q = m/n$



Resonance