Tearing modes

Contents

- Magnetic field diffusion
- Tearing Stability Index
- Rutherford equation

$$\underset{\sim}{E} + \underset{\sim}{u} \times \underset{\sim}{B} = \eta \underset{\sim}{J} \qquad \eta \text{ resistivity } \Omega\text{m}$$

$$\frac{\partial \underset{\sim}{B}}{\partial t} = -\nabla \times \underset{\sim}{E} = \nabla \times (\underset{\sim}{u} \times \underset{\sim}{B}) - \underline{\underline{\nabla \times (\eta \underset{\sim}{J})}}$$

$$\nabla \times (\nabla \times \underline{A}) = \nabla(\nabla \cdot \underline{A}) - \nabla^2 \underline{A}$$

if $\eta = \text{const}$ $\quad \nabla \cdot \underset{\sim}{J} = 0$

$$\frac{\partial \underset{\sim}{B}}{\partial t} = \nabla \times (\underset{\sim}{u} \times \underset{\sim}{B}) - \nabla \times \left(\eta \frac{1}{\mu_0} \nabla \times \underset{\sim}{B}\right)$$

$$- \frac{\eta}{\mu_0} \left[\nabla(\underbrace{\cancel{\nabla \cdot \underset{\sim}{J}}}_{0}) - \nabla^2 \underset{\sim}{B} \right]$$

$$\frac{\partial \underset{\sim}{B}}{\partial t} = \nabla \times (\underset{\sim}{u} \times \underset{\sim}{B}) + \underline{\underline{\frac{\eta}{\mu_0} \nabla^2 \underset{\sim}{B}}} \qquad \left(\frac{\eta}{\mu_0}\right) \text{m}^2/\text{s}$$

Diffusion equation

Collisional plasma

→ Spitzer resistivity $\quad \eta \simeq 5.2 \times 10^{-5} \dfrac{Z^2 \ln \Lambda}{(T_e/\text{eV})^{3/2}} \; \Omega\text{m}$

$T_e \sim 1\,\text{keV}$ $\ln\Lambda \sim 15$ $Z \sim 1$

$\Rightarrow \eta \sim 10^{-8}\,\Omega m$ $\frac{\eta}{\mu_0} \sim 2\times10^{-2}\,m^2/s$

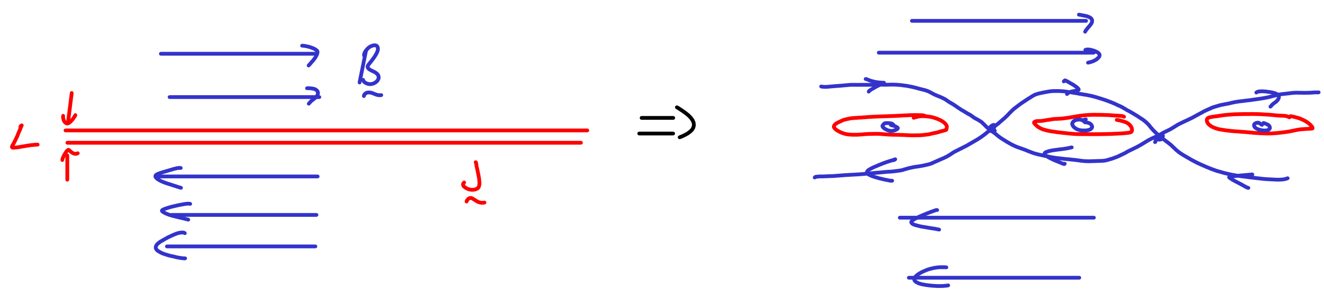
$\nabla^2 \sim \frac{1}{L^2}$ $\frac{\partial}{\partial t} \sim \frac{1}{\tau}$

$\frac{1}{\tau} \sim \frac{\eta}{\mu_0}\frac{1}{L^2}$ $\tau = \frac{L^2\mu_0}{\eta}$

Thin layers $L \sim 1\,mm$ $\tau \sim 50\,\mu s$

$L \sim 1\,m$ $\tau \sim 50\,s$

Eg. slab reconnection



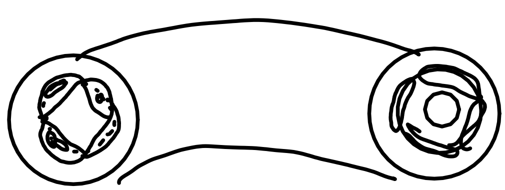
Magnetic islands

Cylindrical Tearing mode equation

$$\delta \underset{\sim}{J}_\phi = \frac{\frac{dJ_\phi}{dr}}{B_\theta[1-qn/m]}\,\delta\psi = 0$$

Singularity at $q = m/n$

$\Rightarrow$ can get tearing modes at $q = m/n$ surfaces



$q = 2/1$

$3/2$

$\Rightarrow$ degrade confinement

Tearing stability index

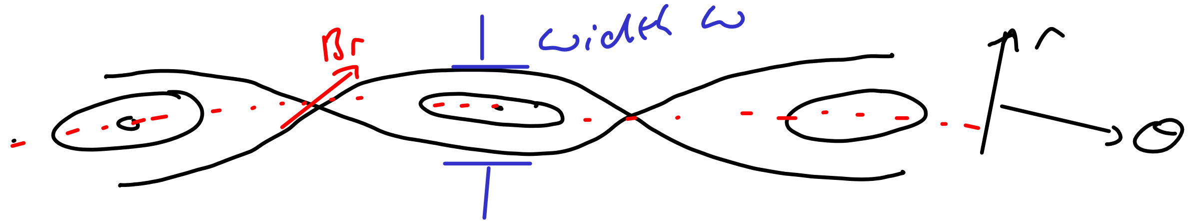
$$\Delta' = \left[\!\left[\frac{1}{\delta\psi} \frac{\partial \delta\psi}{\partial r} \right]\!\right] \quad \text{jump across island}$$

$\sim$ current δJ on surface

$\sim \delta W$

unstable if $\Delta' > 0$

<u>Rutherford equation</u> (1973)



$$\frac{\partial \delta B_r}{\partial t} \simeq \frac{\eta}{\mu_0} \nabla^2 \delta B_r \simeq \frac{\eta}{\mu_0} \frac{\partial^2 \delta B_r}{\partial r^2}$$

$$\delta B_r = -\frac{1}{r} \frac{\partial \delta\psi}{\partial \theta}$$

$$\delta\psi \sim \delta\hat{\psi}(r)\, e^{i(m\theta - n\phi)}$$

$$\delta B_r \simeq -\frac{im}{r} \delta\psi$$

$$\int_{-w/2}^{+w/2} \frac{\partial \delta B_r}{\partial t} dr \simeq \int_{-w/2}^{+w/2} \frac{\eta}{\mu_0} \frac{\partial^2 \delta B_r}{\partial r^2} dr$$

$$\simeq w \frac{\partial \delta B_r}{\partial t} \simeq = \frac{\eta}{\mu_0} \left[\frac{\partial \delta B_r}{\partial r} \right]_{-w/2}^{+w/2}$$

$$\delta B_r \propto w^2 \qquad \frac{dw}{dt} = \frac{\eta}{\mu_0} \left[\frac{1}{\delta\psi} \frac{\partial \delta\psi}{\partial r} \right]_{-w/2}^{w/2} = \frac{\eta}{2\mu_0} \Delta'$$

$$\Delta' > 0 \Rightarrow \frac{dw}{dt} > 0$$