The Ballooning Transform

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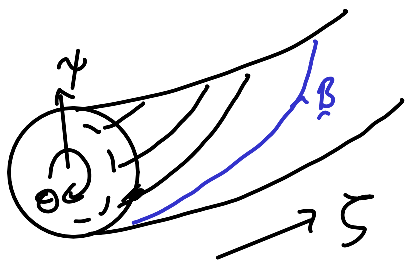


Modes are:

- Elongated along B
- Narrow across B (localised)
- Periodic

$$\phi(\psi, \theta, \zeta) = \phi(\psi, \theta + n2\pi, \zeta + m2\pi)$$

n, m integers

field aligned coordinate system

$$\underset{\sim}{B} = \nabla\alpha \times \nabla\psi \qquad \alpha, \psi \text{ constant along } B$$

$$\alpha = \zeta - q\theta \qquad \theta \text{ varied, move along } B$$

$$\phi(\psi, \theta, \alpha) = \phi(\psi, \theta + n2\pi, \alpha - nq2\pi + m2\pi)$$

$q(\psi)$

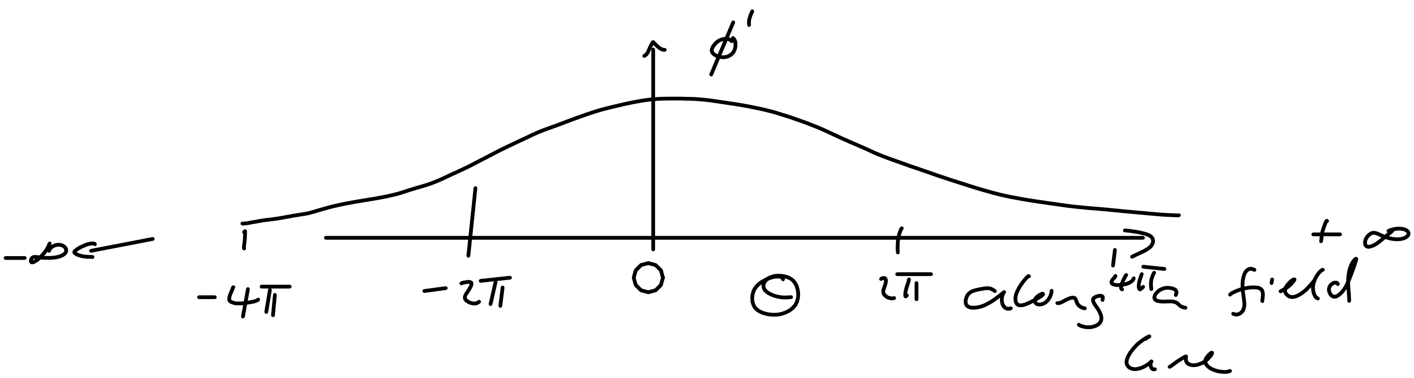
$$\nabla\alpha = \nabla\zeta - q\nabla\theta - \theta q' \nabla\psi$$

$$\theta \rightarrow \theta + 2\pi \qquad \nabla\alpha \rightarrow \nabla\alpha - 2\pi q' \nabla\psi$$

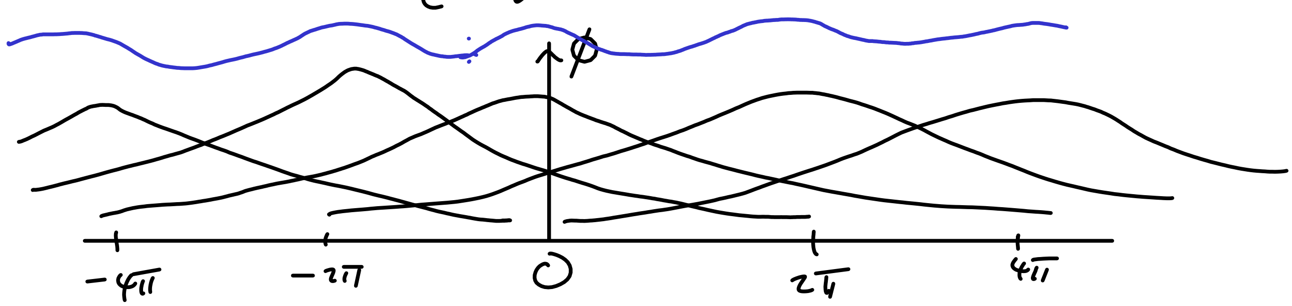
$\Rightarrow$ hard to enforce periodicity

Ballooning transform

Extend $0 < \theta < 2\pi$ to $-\infty < \theta < \infty$



$$\phi(\psi, \theta, \alpha) = \sum_{l=-\infty}^{\infty} \phi'(\psi, \theta + l2\pi, \alpha - 2\pi q l)$$



✓ periodic in θ, ζ

✓ elongated along B

Fourier modes

$$\phi(\psi, \theta, \alpha) = \sum_{l=-\infty}^{\infty} \phi'(\psi, \theta + 2\pi l, \alpha - 2\pi q l)$$

$$= \sum_{l=-\infty}^{\infty} \underbrace{\phi'(\psi, \theta + 2\pi l)}_{\text{slowly varying}} \underbrace{e^{-in(\alpha - 2\pi q l)}}_{\text{fast varying}}$$

↖ toroidal mode number

n large (high n)

$$\nabla\phi'(\psi,\theta+2\pi\ell,\alpha-2\pi q\ell) = -in\,\phi'(\psi,\theta+2\pi\ell)\,(\nabla\alpha - 2\pi\ell q'\nabla\psi) \times e^{-in(\alpha-2\pi q\ell)}$$

$$\nabla\alpha = \nabla\zeta - q\nabla\theta - \theta q'\nabla\psi$$

$$\Rightarrow \nabla\phi'(\psi,\overbrace{\theta+2\pi\ell},\alpha-2\pi q\ell) = -in\,\phi'(\psi,\overbrace{\theta+2\pi\ell})\,\overline{\nabla(\zeta - q(\overbrace{\theta+2\pi\ell}^{\theta}))}^{\alpha} \times e^{-in(\alpha-2\pi q\ell)}$$

$$\nabla\phi(\psi,\theta,\alpha) = -in\nabla\alpha\,\phi(\psi,\theta,\alpha)$$ perp. gradient

$$\underset{\sim}{B}\cdot\nabla\phi = \frac{1}{J}\frac{\partial\phi}{\partial\theta}$$ parallel gradient

Ballooning equation (from SW)

$$\underset{\sim}{B}\cdot\nabla\left[\frac{1}{\mu_0 B^2}\nabla^2(\underset{\sim}{B}\cdot\nabla\phi)\right] + 2\frac{\underset{\sim}{\kappa}\times\underset{\sim}{B}}{B^2}\cdot\nabla\left(\frac{\nabla p\times\underset{\sim}{B}}{B^2}\cdot\nabla\phi\right) = 0$$

$$\frac{\overbrace{\nabla p}^{p'\nabla\psi}\times\underset{\sim}{B}}{B^2}\cdot\nabla\alpha = \frac{p'}{B^2}\underbrace{(\nabla\alpha\times\nabla\psi)}_{\underset{\sim}{B}}\cdot\underset{\sim}{B} = p'$$

$$\frac{\underset{\sim}{\kappa}\times\underset{\sim}{B}}{B^2}\cdot\nabla\alpha = \frac{1}{B^2}(\underset{\sim}{B}\times\nabla\alpha)\cdot\underset{\sim}{\kappa} = \frac{1}{B^2}\Big(\underset{\sim}{B}\times\overbrace{(\nabla\zeta - q\nabla\theta)}^{\underset{\sim}{s}} - \theta q'\underset{\sim}{B}\times\nabla\psi\Big)\cdot\underset{\sim}{\kappa}$$

$$\underset{\sim}{\kappa} = (\underset{\sim}{b}\cdot\nabla)\underset{\sim}{b}$$

normal

$$\underset{\sim}{\kappa} = \kappa_\psi\nabla\psi + \kappa_s\underset{\sim}{s}$$

geodesic

$$(\underset{\sim}{B}\times\nabla\psi)\cdot\underset{\sim}{\kappa} = -B^2\kappa_s$$

$$(\underset{\sim}{B}\times\underset{\sim}{s})\cdot\underset{\sim}{\kappa} = B^2\kappa_\psi$$

$$\frac{\kappa \times B}{B^2} \cdot \nabla \alpha = \kappa_\psi + \theta q' \kappa_s$$

Ballooning equation

$$\frac{1}{J}\frac{\partial}{\partial \theta}\left[\frac{|\nabla \alpha|^2}{\mu_0 B^2}\frac{1}{J}\frac{\partial \phi}{\partial \theta}\right] + 2p'\left(\kappa_\psi + \theta q' \kappa_s\right)\phi = 0$$

1D ϕ depends on θ

solve on each flux surface separately

$\rightarrow$ calculate high n stability