MHD shocks

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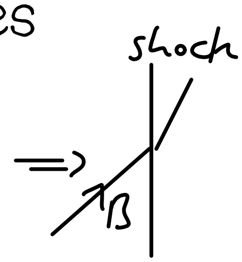
not covering oblique shock

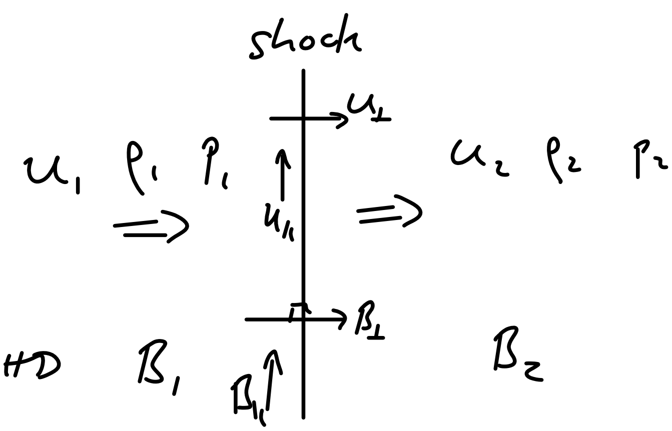


v. complicated

MHD has 3 waves:

Alfven, fast & slow magnetosonic

$\Rightarrow$ More different regimes depending on flow speed relative to these three wave speeds.



In MHD

Start from conservation form of MHD

Mass $\quad \cancel{\dfrac{\partial \rho}{\partial t}} + \nabla \cdot \underbrace{(\rho \underline{u})}_{\text{const}} = 0$

$$\text{Momentum} \quad \cancel{\frac{\partial (\rho \underline{u})}{\partial t}} + \nabla \cdot \underbrace{\left[\rho \underline{u}\,\underline{u} + \left(p + \frac{B^2}{2\mu_0}\right)\mathbb{I} - \frac{\underline{B}\,\underline{B}}{\mu_0} \right]}_{\text{Const}} = 0$$

MHD

Energy
$$\cancel{\frac{\partial E}{\partial t}} + \nabla\cdot\Big[\underbrace{\left(\tfrac{1}{2}\rho u^2 + \tfrac{\gamma}{\gamma-1}p\right)\underline{u} + \frac{\underline{B}\times(\underline{u}\times\underline{B})}{\mu_0}}_{\text{const}} \Big] = 0$$

rest-frame of shock

$$E = \tfrac{1}{2}\rho u^2 + \frac{1}{\gamma-1}p + \frac{B^2}{2\mu_0}$$

$$\nabla\cdot\underline{B} = 0 \qquad \cancel{\frac{\partial \underline{B}}{\partial t}} = \nabla\times(\underline{u}\times\underline{B})$$

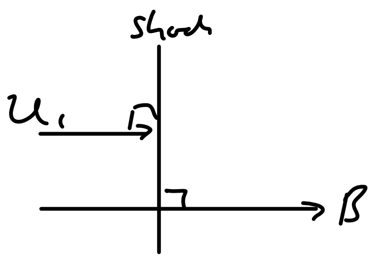
$$B_\perp = \text{const}$$

This reduces to the MHD Rankine-Hugoniot equations

$$\boxed{\begin{aligned}
&B_\perp = \text{const} && \rho u_\perp^2 + p + \frac{B_\parallel^2}{2\mu_0} = 0\\
&u_\perp B_\parallel - u_\parallel B_\perp = \text{const} && \rho u_\perp u_\parallel - \frac{B_\perp B_\parallel}{\mu_0} = \text{const}\\
&\rho u_\perp = \text{const} &&\\
&\tfrac{1}{2}\rho u^2 u_\perp + \frac{\gamma}{\gamma-1} p u_\perp + \frac{B_\parallel(u_\perp B_\parallel - u_\parallel B_\perp)}{\mu_0} = \text{const} &&
\end{aligned}}$$

6 equations, 6 unknowns $\rho_2, P_2 \quad u_\parallel \quad u_\perp \quad B_\parallel \quad B_\perp$

① Parallel (Hydrodynamic) shock



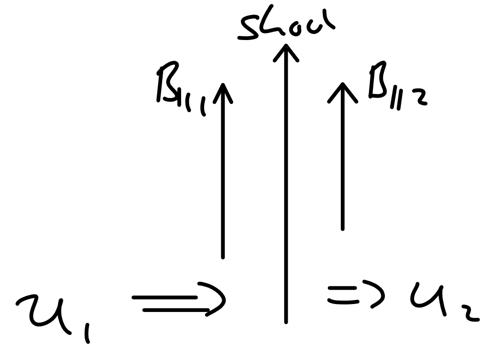
$$B_\parallel = 0 \qquad u_\parallel = 0$$

$\Rightarrow$ Same as fluid $(B=0)$ case

$$M_1 > 1 \qquad M_2 < 1$$

Shock moves > sound speed into fluid.

② Perpendicular shock



$u_\parallel = 0$

$B_\perp = 0$

$\rho u_\perp = \text{const}$ $\qquad \dfrac{\rho_2}{\rho_1} = \dfrac{u_1}{u_2}$ $\qquad \overset{B_\parallel}{\dfrac{B_2}{B_1}} = \overset{u_\perp}{\dfrac{u_1}{u_2}}$

$\dfrac{B_2}{B_1} = \dfrac{\rho_2}{\rho_1}$ Magnetic field increased by same ratio as the density.

$u_1^2 > C_s^2 + V_A^2$ Fast magnetosonic wave

Discontinuities

$u_\perp = 0$ no flow across discontinuity

Important for fluid boundaries
boundary layers

$B_\perp = \text{const}$

$p + \dfrac{B^2}{2\mu_0} = \text{const}$

$\dfrac{B_\perp B_\parallel}{\mu_0} = \text{const}$ $\qquad u_\parallel \dfrac{B_\parallel B_\perp}{\mu_0} = \text{const}$

Contact discontinuity

$$B_\perp \neq 0$$

$$\Rightarrow B_\parallel \text{ and } u_\parallel \text{ constant}$$

No change in tangential flow or B field across a contact discontinuity

(different from hydrodynamic case)

Tangential discontinuity

$$B_\perp = 0$$

$\Rightarrow$ only equation is $\quad p + \frac{B^2}{2\mu_0} = \text{const}$

(force balance)

e.g. plasma-vacuum boundary

can have arbitrary jumps in $u_\parallel$, ρ, p, $B_\parallel$