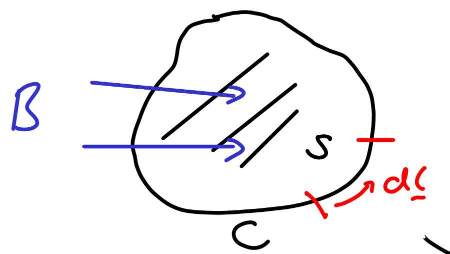
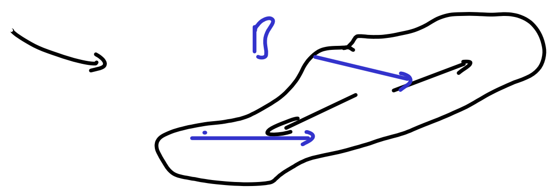
Frozen Flux Theorem

Contents

- Proof
- Magnetic field topology
- Flux tubes
- Magnetic field velocity



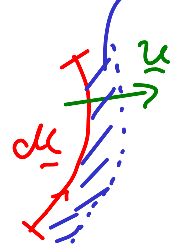
$$\Phi = \int_S \underline{B} \cdot d\underline{S} \qquad \text{flux}$$



$$\frac{d\Phi}{dt} = \int \frac{\partial \underline{B}}{\partial t} \cdot dS + \int \underline{B} \cdot \frac{\partial d\underline{S}}{\partial t}$$

$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{u} \times \underline{B})$

change in S

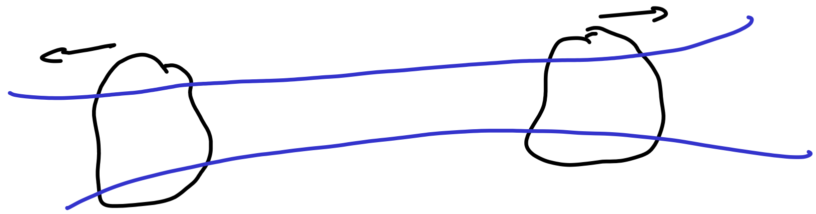


$$\Delta S = \Delta t\, \underline{u} \times d\underline{l}$$

$$\frac{d\Phi}{dt} = \int \nabla \times (\underline{u} \times \underline{B}) \cdot d\underline{S} + \int_C \underline{B} \cdot (\underline{u} \times d\underline{l})$$

Stoke's theorem

$$\frac{d\Phi}{dt} = \int_C (\underline{u} \times B) \cdot d\underline{l} + \underbrace{\int_C \underline{B} \cdot (\underline{u} \times d\underline{l})}_{\int_C -(\underline{u} \times \underline{B}) \cdot d\underline{l}} = 0$$



"flux tube"

→ field line

closed loops remain linked along the magnetic field as the plasma moves.

Non-ideal

$$\underset{\sim}{E} + \underset{\sim}{w} \times \underset{\sim}{B} = 0 \quad \Rightarrow \text{field frozen}$$

↑ field line velocity

ideal MHD $\underset{\sim}{w} = \underset{\sim}{u}$ fluid

Hall MHD $\underset{\sim}{w} = \underset{\sim}{u}_e$ electron fluid

in general not well defined