Dissipation and Ideal MHD

Contents

- Limits of ideal MHD
- Missing physics
- Layer models

Assuming Collision dominated, so close to Maxwellian
- neglects the effect of that dissipation
- Fusion plasmas are nearly collisionless

Empirically Ideal MHD is found to work well

Because
1) The parallel dynamics which the model gets wrong are usually not important for equilibrium and large-scale instabilities
2) The perpendicular force balance is captured quite accurately by Ideal MHD

Some missing physics:
1) Viscosity, due to collisions or Finite Larmor Radius
2) Resistivity
3) Wave-particle resonances
   - Landau closure (Landau damping)

Viscosity

Ion momentum

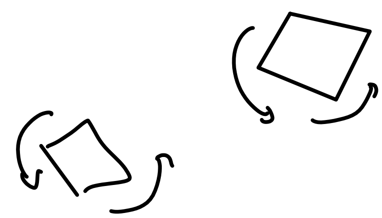
$Tr[\Pi] = 0$

$$\rho\left[\frac{\partial \underset{\sim}{u}}{\partial t} + \underset{\sim}{u}\cdot\nabla \underset{\sim}{u}\right] = \underset{\sim}{J}\times \underset{\sim}{B} - \nabla p - \nabla\cdot\underline{\underline{\Pi}}$$

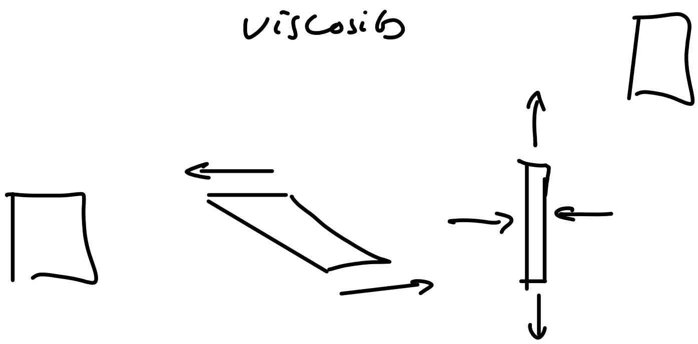
Stress tensor (viscosity)

$$-\nabla\cdot\underline{\underline{\Pi}} = \mu\nabla^2 \underset{\sim}{u} + \frac{1}{3}\mu\nabla(\nabla\cdot\underset{\sim}{u})$$

dynamic viscosity



$\nabla\cdot\Pi = 0$ under translation or rotation



$\nabla\cdot\underline{\underline{\Pi}} \neq 0$

μ dynamic $= \rho\nu$

ν kinematic $[m^2/s]$

Normalise u_0 speed

L length

L/u_0 time

$\hat{\underset{\sim}{u}} = \underset{\sim}{u}/u_0$ $\quad \hat{\nabla} = \nabla L$

$$\underbrace{\rho \frac{u_0^2}{L}}\left[\frac{\partial \hat{\underline{u}}}{\partial \hat{t}} + \hat{\underline{u}} \cdot \hat{\nabla} \hat{\underline{u}} \right] = \cdots\cdots \underbrace{\overset{\rho\nu}{\mu} \frac{u_0}{L^2}} \left(\hat{\nabla}^2 \underline{u} + \frac{1}{3} \hat{\nabla}(\hat{\nabla}\cdot\underline{u}) \right)$$

$$\frac{\partial \hat{\underline{u}}}{\partial \hat{t}} + \hat{\underline{u}} \cdot \hat{\nabla} \hat{\underline{u}} = \cdots + \underbrace{\frac{\nu}{u_0 L}}_{\frac{1}{Re}} \left(\hat{\nabla}^2 \underline{u} + \frac{1}{3} \hat{\nabla}(\hat{\nabla}\cdot\underline{u}) \right)$$

$$Re = \frac{u_0 L}{\nu}$$

$$Re = \frac{\text{viscous diffusion time}}{\text{advection time}}$$

Resistivity

$$\underline{E} + \underline{u} \times \underline{B} = \eta \underline{j}$$

note: $\eta_\perp \simeq 1.96\,\eta_\parallel$

$$\frac{\partial \underline{B}}{\partial t} \doteq \nabla \times (\underline{u} \times \underline{B}) - \underbrace{\nabla \times (\eta \underline{j})}_{\simeq -\underbrace{\frac{\eta}{\mu_0}}_{\text{Diffusion coefficient}} \nabla^2 \underline{B}}$$

$$\underline{j} = \frac{1}{\mu_0} \nabla \times \underline{B}$$

Typical length L

speed u_0

Magnetic field B_0

$$\underbrace{\frac{B_0 u_0}{L}} \frac{\partial \hat{\underline{B}}}{\partial \hat{t}} = \underbrace{\frac{B_0 u_0}{L}} \hat{\nabla} \times (\hat{\underline{u}} \times \hat{\underline{B}}) + \frac{\eta}{\mu_0} \frac{1}{L^2} B_0 \hat{\nabla}^2 \hat{\underline{B}}$$

$$\frac{\partial \hat{B}}{\partial \hat{t}} = \hat{\nabla} \times (\hat{\underline{u}} \times \hat{\underline{B}}) + \underbrace{\frac{\eta}{\mu_0 u_0 L}}_{\frac{1}{R_m}} \hat{\nabla}^2 \hat{\underline{B}}$$

Magnetic Reynolds number

$$R_m = \frac{\text{Resistive diffusion timescale}}{\text{advection timescale}}$$

Set $u_0 = V_A$ Alfvén speed

$$S = \frac{\mu_0 V_A L}{\eta}$$

Lundquist number

Small scales and Lengths

- if $R_e \sim 1$ viscosity ~ advection

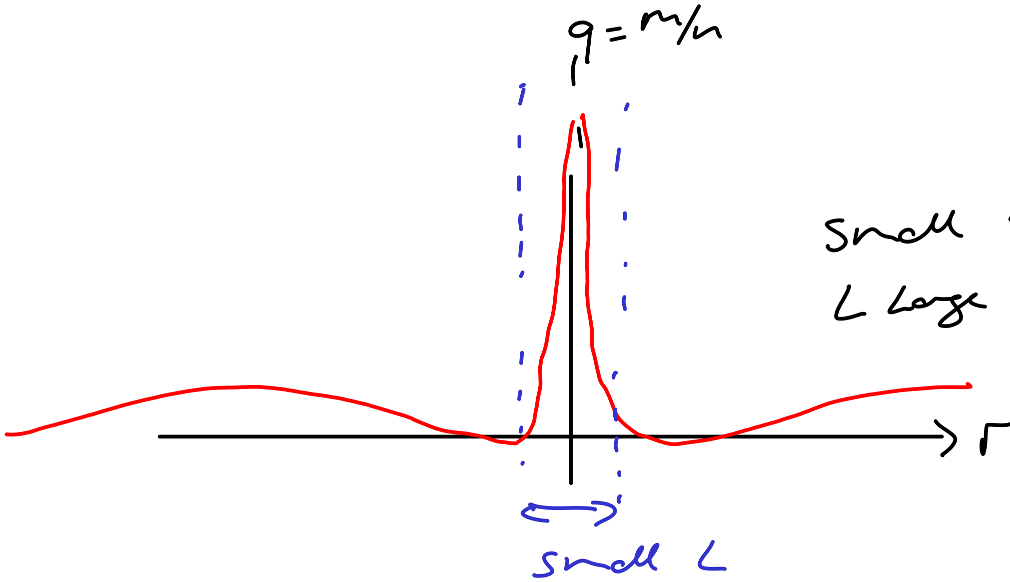
$$\frac{u_0 L}{\nu} \sim 1 \qquad \underline{L_\nu \sim \frac{\nu}{u_0}}$$

- If $R_m \sim 1$ resistive ~ advection

$$\frac{\mu_0 L u_0}{\eta} \sim 1 \qquad \underline{L_\eta \sim \frac{\eta}{\mu_0 u_0}}$$

if $L \gg L_\nu, L_\eta$ => dissipation is small ~ ideal MHD

if $L \sim L_\nu, L_\eta$ => non-ideal effects important



small gradients
L large → ideal

non-ideal
case