Plasma resistivity

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- Linearised collision operator
- Lorentz operator
- Solution using Legendre polynomials

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$$\eta = \frac{\pi Z e^2 m^{1/2} \ln\Lambda}{(4\pi\varepsilon_0)^2 \, (eT)^{3/2}}$$

$\ln\Lambda$ ← Coulomb Log ~ 16

T ← T in eV

velocity increases → lower collision rate

Resistivity

A linear relationship between electric field and current density

$$\underline{\underline{E}} = \eta \, \underline{J} \qquad (\eta \text{ tensor or scalar})$$

Starting from Vlasov + collisions (electron)

$$\frac{\partial f_e}{\partial t} + \underline{v}\cdot\nabla f_e - \frac{e}{m_e}(\underline{E} + \underline{v}\times\underline{B})\cdot\frac{\partial f_e}{\partial \underline{v}} = C_e(f_e)$$

velocity coordinate

collision

$f_e(t, \underset{\sim}{x}, \underset{\sim}{v})$ electron distribution function

for homogeneous, long timescale, along $\underset{\sim}{B}$

$$-\frac{e}{m} \underset{\sim}{E} \cdot \frac{\partial f_e}{\partial \underset{\sim}{v}} = C_e(f_e) \qquad (1)$$

$$\underset{\sim}{J} = -e \int \underset{\sim}{v} f_e \, d\underset{\sim}{v}$$

$\rightarrow$ nonlinear equation for $\underset{\sim}{J}, \underset{\sim}{E}$

$$C_e(f_e) \sim f_e \nu_e$$

$$-\frac{e}{m_e} E \frac{\cancel{f_e}}{v_{Te}} \sim \cancel{f_e} \nu_e \qquad E \sim E_R = \frac{m_e v_{Te} \nu_e}{e}$$

thermal speed

balance collisions against acceleration by E field

If $E \gtrsim E_R$ then electrons accelerate

$\rightarrow$ collision rate goes down as v increases

$\rightarrow$ Further acceleration to high energy

$\Rightarrow$ Runaway fluid models break

<u>Assume</u> $E \ll E_R$ $\qquad E/E_R \simeq \varepsilon$ small parameter

perturbation $f_e = f_e^0 + f_e^1 + f_e^2 + \cdots$

Zeroth order

$$0 = C(f_e^0)$$

H theorem

f_e^0 is Maxwellian

$$f_e^0(\underline{v}) = \frac{n}{(2\pi v_{Te}^2)^{3/2}} e^{-|\underline{v}-\underline{u}|^2/v_{Te}^2}$$

fluid velocity

First order

$$-\frac{e}{m_e} \underset{\sim}{E} \cdot \frac{\partial}{\partial v} f_e^0 = C_e(f_e^1)$$

$$C_e(f_e^1) = \frac{2e}{m_e v_{Te}^2} \underset{\sim}{E} \cdot \underset{\sim}{v} f_e^0$$

$$C_e(f_e^1) = C_{ei}(f_e, f_i) + C_{ee}(f_e)$$

electron-ion

$m_i \to \infty$

Lorentz limit

electron-electron

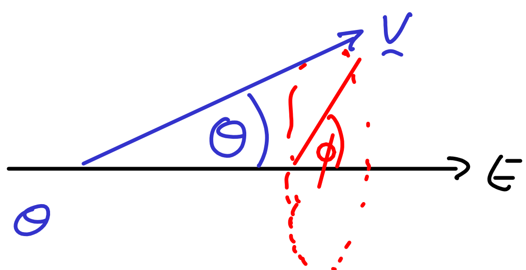
$C_{ee}(f_e^0, f_e^1)$

$+ C_{ee}(f_e^1, f_e^0)$

linearised

To illustrate the procedure

Consider C_{ei} only



Azimuthal angle ϕ

pitch angle θ

$$\underset{\sim}{E}\cdot\underset{\sim}{v} = E v \underbrace{\cos\theta}_{\mu}$$ pitch angle variable

$$\frac{2e}{m_e v_{Te}^2} E v \mu f_e^0 = \nu_{ei}\frac{v_{Te}^3}{v^3}\left[\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial}{\partial\mu} + \cancel{\frac{1}{1-\mu^2}\frac{\partial^2}{\partial\phi^2}}\right] f_e^1$$

indep. of ϕ

Use Legendre Polynomials

$$f_e^1(v,\mu) = \sum_\ell a_\ell(v) P_\ell(\mu)$$

$$\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial}{\partial\mu} P_\ell(\mu) = -\ell(\ell+1) P_\ell(\mu)$$

$P_1(\mu) = \mu \quad \Rightarrow$ only one term $\quad a_1(v) P_1(\mu) = f_e^1$

$$f_e^1 = -2\frac{E}{E_R}\left(\frac{v}{v_{Te}}\right)^4 \mu f_e^0$$

Tails of distribution function important

$$\underset{\sim}{J} = \frac{e v_{Te} n}{E_r}\frac{8}{\sqrt{\pi}} E$$

$$\Rightarrow \eta^L = \frac{\sqrt{\pi}}{8}\frac{m_e \nu_{ei}}{e^2 n}$$

$$\eta^{SH} = \frac{2}{3\sqrt{\pi}} \frac{m_e \nu_{ei}}{e^2 n}$$

$\uparrow$

factor of ~ 1.7 larger

Including electron-electron collisions

$\longrightarrow$ Towards Maxwellian

$\longrightarrow$ Increases average collision rate

$$\nu_{ei} \sim \frac{1}{v^3}$$