Ion viscosity

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} Collisional

← collisionless

$$\underline{\underline{P}} = p\underbrace{\underline{\underline{I}}}_{\delta_{\alpha\beta}} + \underbrace{\underline{\underline{\Pi}}}_{\Pi_{\alpha\beta}}$$

$P_{\alpha\beta}$

Braginskii $\quad \underline{\underline{\Pi}} = \sum_{i=0}^{4} \Pi_i$

<u>Parallel viscosity</u>

B field unit vector

$$\Pi_0 = -3\eta_0 \left(\underset{\sim}{b}\underset{\sim}{b} - \tfrac{1}{3}\underline{\underline{I}}\right)\underbrace{\left(\underset{\sim}{b}\underset{\sim}{b} - \tfrac{1}{3}\underline{\underline{I}}\right)}_{b_\alpha b_\beta - \frac{1}{3}\delta_{\alpha\beta}} : \nabla \underset{\sim}{u}$$

$\partial_\alpha u_\beta$

If $\nabla \underset{\sim}{b} = 0$

$$\left(\underset{\sim}{b}\underset{\sim}{b} - \tfrac{1}{3}\underline{\underline{I}}\right) : \nabla\underset{\sim}{u} = \underbrace{(\underset{\sim}{b}\cdot\nabla)(\underset{\sim}{b}\cdot\underset{\sim}{u})}_{\partial_\parallel u_\parallel} - \tfrac{1}{3}\nabla\cdot\underset{\sim}{u}$$

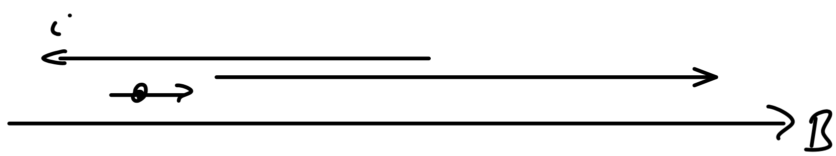
$\nabla\cdot\underline{\underline{\Pi}}$ contain $\sim \eta_0 \partial_\parallel(\partial_\parallel u_\parallel)$

Diffusion of $u_\parallel$ along $\underset{\sim}{b}$

$$\eta_0^i = 0.96\, e n T_i \tau_i \quad \xleftarrow{} \ 1/\nu_i \text{, collision time}$$

$$\eta_0^e = 0.73\, e n T_e \tau_e$$

fewer collisions $\rightarrow \eta_0$ increases



Perpendicular viscosity

$$\Pi_1 = -\eta_1 \left[\underline{\underline{I}}_\perp \cdot \underline{\underline{W}} \cdot \underline{\underline{I}}_\perp + \tfrac{1}{2} \underline{\underline{I}}_\perp (\underset{\sim}{b} \cdot \underline{\underline{W}} \cdot \underset{\sim}{b}) \right]$$

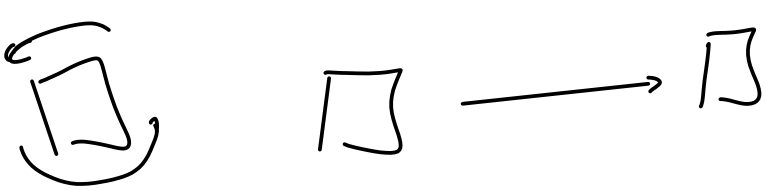
$$\Pi_2 = -4\eta_1 \left[\underline{\underline{I}}_\perp \cdot \underline{\underline{W}} \cdot \underset{\sim}{b}\underset{\sim}{b} + \underset{\sim}{b}\underset{\sim}{b} \cdot \underline{\underline{W}} \cdot \underline{\underline{I}}_\perp \right]$$

$$\underline{\underline{I}}_\perp = \underline{\underline{I}} - \underset{\sim}{b}\underset{\sim}{b}$$

$\underline{\underline{W}}$ is the rate of strain tensor

$$W_{\alpha\beta} = \frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha} - \frac{2}{3} (\nabla \cdot \underset{\sim}{u}) \delta_{\alpha\beta}$$

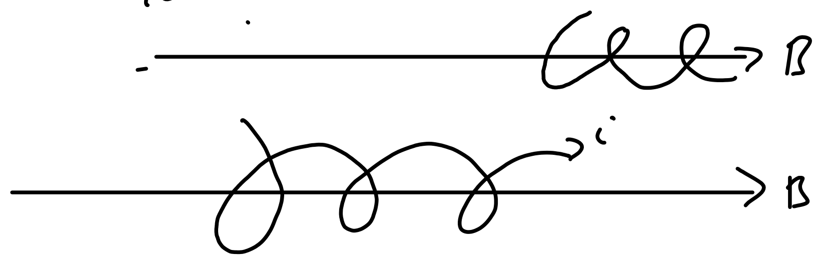
Zero under translation or rotation



Measures deformation of fluid element.

$$\eta_1^i = \frac{3 n e T_i}{10\, \Omega_i^2 \tau_i} \qquad \eta_1^e = 0.51 \frac{n e T_e}{\Omega_e^2 \tau_e}$$

As collisions become less frequent, η_1 gets smaller (opposite to η_0)



Transfer of momentum across the magnetic field smaller than parallel viscosity by $\sim (\rho/L_\parallel)^2$

Gyro-viscosity

$$\Pi_3 = \frac{\eta_3}{2}\left[\underset{\sim}{b}\times\underline{\underline{W}}\cdot\underline{\underline{I}}_\perp - \underline{\underline{I}}_\perp\cdot\underline{\underline{W}}\times\underset{\sim}{b}\right]$$

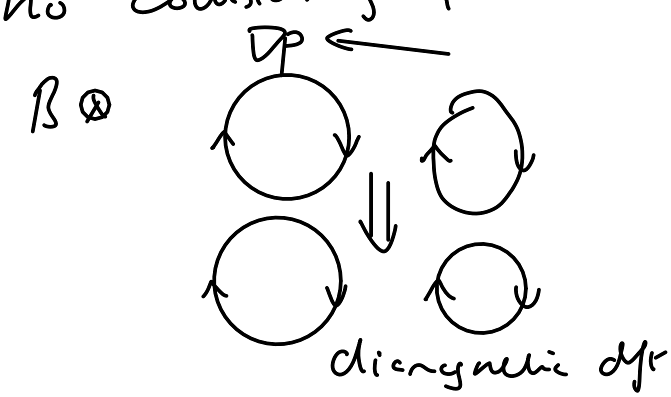
$$\Pi_4 = 2\eta_3\left[\underset{\sim}{b}\times\underline{\underline{W}}\cdot\underset{\sim}{b}\underset{\sim}{b} - \underset{\sim}{b}\underset{\sim}{b}\cdot\underline{\underline{W}}\times\underset{\sim}{b}\right]$$

Perpendicular to B and to velocity

$\Rightarrow$ no dissipation (viscous heating)

$$\eta_3^i = \frac{enT_i}{2\Omega_i} \qquad \eta_3^e = \frac{-enT_e}{2|\Omega_e|}$$

no collision frequency dependence



To lowest order this gyroviscosity cancels the diamagnetic drift in momentum equation

"The Gyroviscous Cancellation"

$$u_* \cdot \nabla \underset{\sim}{u} \simeq -\nabla \cdot \Pi_g \qquad \Pi_g = \Pi_3 + \Pi_4$$

gyroviscous terms

often assumed, not exact

$$\rho \left[\frac{\partial \underset{\sim}{u}}{\partial t} + (\underset{\sim}{u} - u_*) \cdot \nabla \underset{\sim}{u} \right] = -\nabla p + \underset{\sim}{J} \times \underset{\sim}{B} - \nabla \cdot \overline{\Pi}$$