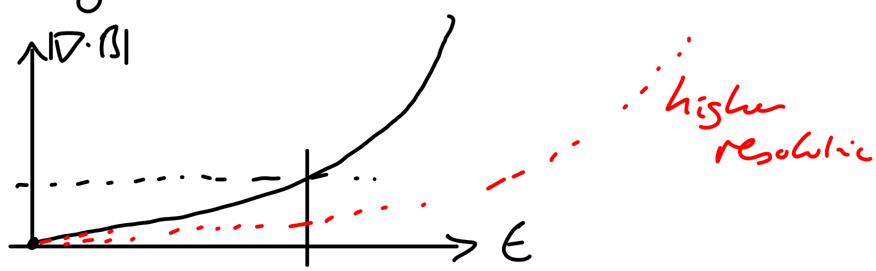
Preserving Div B = 0

- Constraints on evolution equations
- Vector potential
- Divergence cleaning
- Constrained transport

2 approaches:

- Use as diagnostic - calculate $\nabla \cdot \underline{B}$



- Ensure $|\nabla \cdot B| < \varepsilon_{tol}$ tolerance

  - can no longer use as a diagnostic

1) Vector potential

$$\frac{\partial \underline{B}}{\partial t} + \nabla \times (\underline{B} \times \underline{u}) = 0$$

$$\underline{B} = \nabla \times \underline{A} \qquad \frac{\partial \underline{A}}{\partial t} + \underline{B} \times \underline{u} = 0$$

vector potential

Calculate $\underline{B}$ from $\nabla \times \underline{A}$ at every time step

2) Divergence cleaning

- Spread local errors in $\nabla \cdot \underline{B}$, remove from domain through advection and diffusion.

$$\frac{\partial B}{\partial t} + \nabla \times (\underline{B} \times \underline{u}) = 0$$

$$\frac{\partial \underline{B}}{\partial t} + \nabla \cdot (\underline{u}\underline{B} - \underline{B}\underline{u}) = 0 \qquad \frac{\partial B_i}{\partial t} + \partial_j (u_j B_i - B_j u_i) = 0$$

add term

$$\frac{\partial \underline{B}}{\partial t} + \nabla \times (\underline{B} \times \underline{u}) + \nabla \psi = 0$$

$$\frac{1}{c_h^2} \frac{\partial \psi}{\partial t} + \frac{1}{c_p} \psi + \nabla \cdot \underline{B} = 0$$

$$\frac{\partial}{\partial t} \nabla \cdot \underline{B} + \nabla \cdot \nabla \psi = 0$$

$$\frac{\partial^2}{\partial t}(\nabla \cdot \underline{B}) = -\nabla^2 \frac{\partial \psi}{\partial t} \qquad = c_h^2 \nabla^2 (\nabla \cdot \underline{B}) \quad \text{if } c_p \to \infty$$

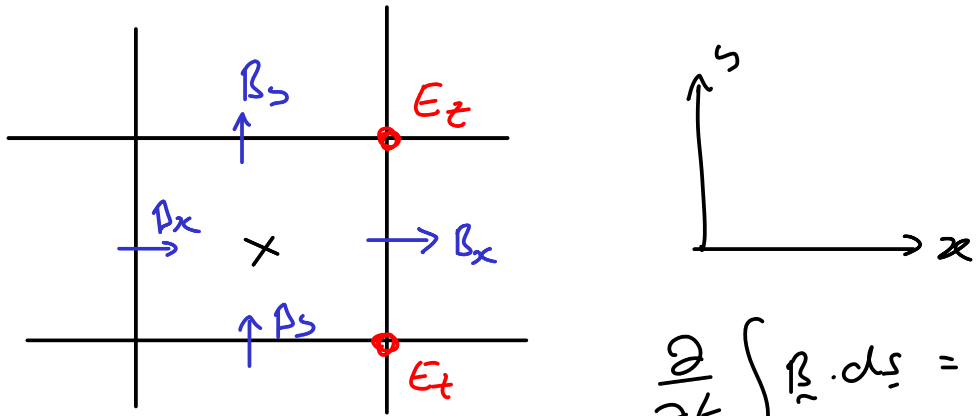
wave equation
speed c_h

$$\frac{\partial}{\partial t}(\nabla \cdot \underline{B}) = c_p^2 \nabla^2 (\nabla \cdot \underline{B}) \quad \text{if } c_h \to \infty \qquad \textbf{Diffusion}$$

Combination of diffusion and advection of $\nabla \cdot \underline{B}$

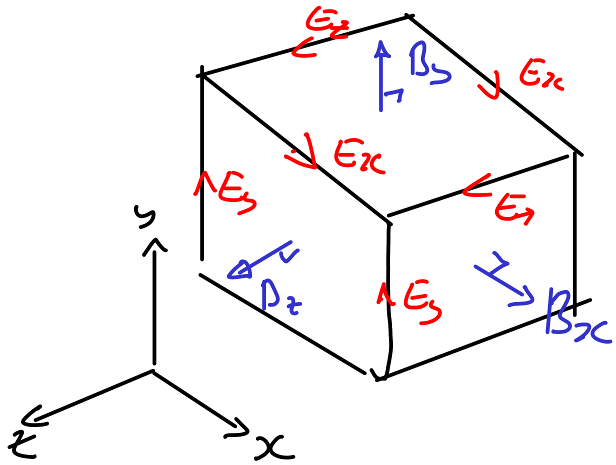
3) Constrained Transport

S. Fromang et. al. A&A 457 : 371-384 (2006)
arXiv:astro-ph/0607230



$$\frac{\partial}{\partial t}\int \underline{B}\cdot d\underline{s} = \oint \underline{E}\cdot d\underline{l}$$

B field on surfaces
E field on edges



$\Rightarrow \nabla\cdot\underline{B} = 0$ preserved to machine precision

see: Yee cell
FDTD

- need interpolations to cell centre
- Introduces error in energy conservation