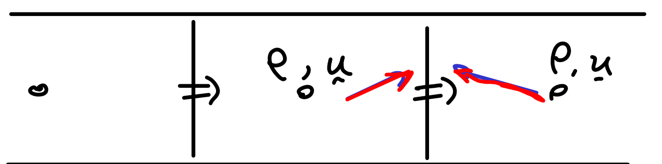
Staggered schemes

Contents

- Advantages and disadvantages
- Dispersion relation

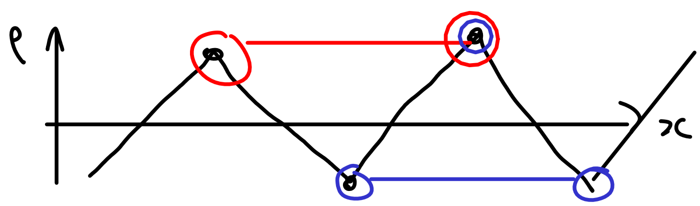
① Cell centred (Collocated)



\+ nonlinear terms easy to calculate

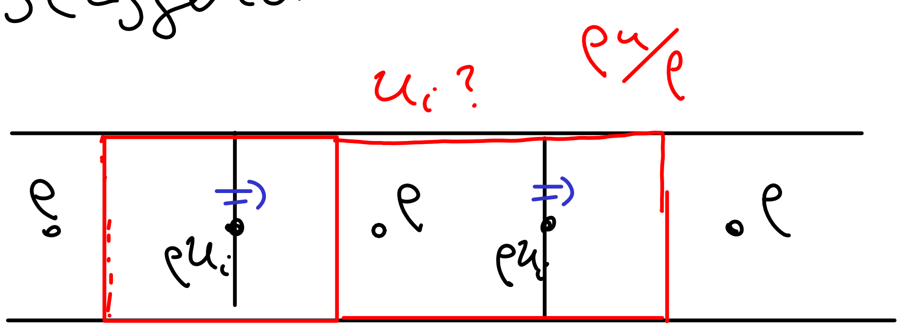
\- need to map from cell centre to edges

\- Zig-zag modes



$\frac{\partial \rho}{\partial x} \simeq 0$

② Staggered

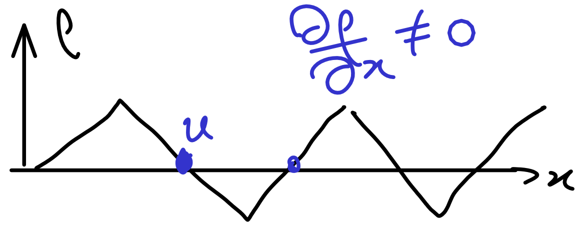


\+ Easier to calculate fluxes

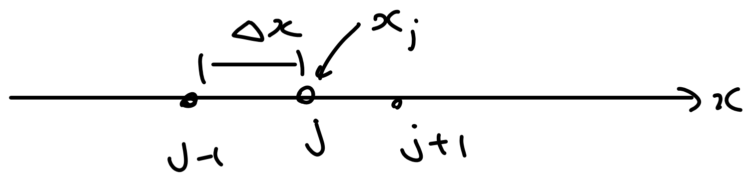
\- Nonlinear terms on different locations

- Conservation requires two sets of grids

+ Better dispersion relation



Dispersion relation



$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} \qquad \frac{\partial p}{\partial t} = -\frac{\partial u}{\partial x}$$

wave $c_s = 1$ $\qquad \underline{\underline{\omega = k}}$

① centred central differencing

$$\frac{\partial p_j}{\partial x} \simeq \frac{p_{j+1} - p_{j-1}}{2\Delta x}$$

$$p(x_j, t) = p_0 \, e^{i(kx_j - \omega t)} \qquad \text{wave}$$

$$p(x_{j+1}, t) = p_0 \, e^{i(kx_j - \omega t)} \cdot e^{ik\Delta x}$$

$$\frac{\partial p_j}{\partial x} \simeq p_0 \, e^{i(kx_j - \omega t)} \underbrace{\frac{\left(e^{ik\Delta x} - e^{-ik\Delta x}\right)}{2\Delta x}}_{2i\sin(k\Delta x)/2\Delta x}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

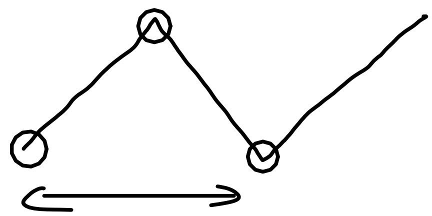
$$-i\omega u_0 = -p_0 \, 2i \frac{\sin(k\Delta x)}{2\Delta x}$$

$$-i\omega p_0 = -u_0 \, i \frac{\sin(k\Delta x)}{\Delta x}$$

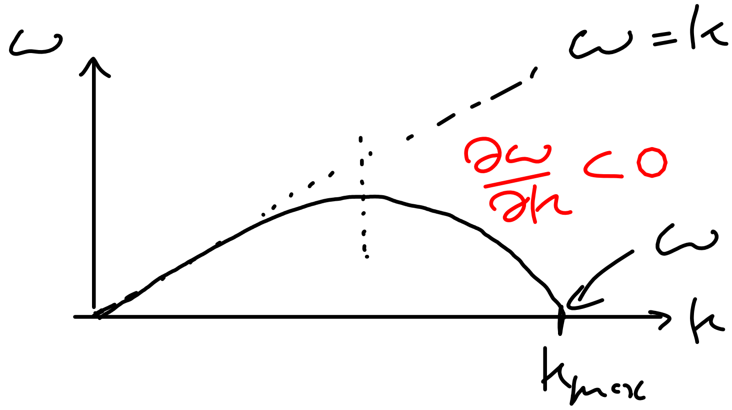
$$\boxed{\omega^2 = \frac{\sin^2(k\Delta x)}{\Delta x^2}}$$

$$k_{max} = \frac{2\pi}{\lambda_{min}}$$

$$= \pi/\Delta x$$



$2\Delta x$



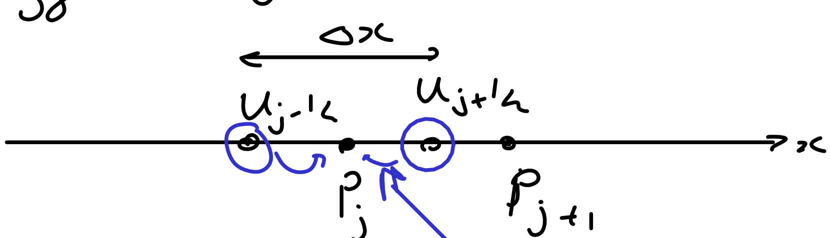
$\frac{\partial\omega}{\partial k} < 0$

parasitic waves

$\omega = 0$ for $\lambda = 2\Delta x$

unphysical high-k waves

② staggered grids



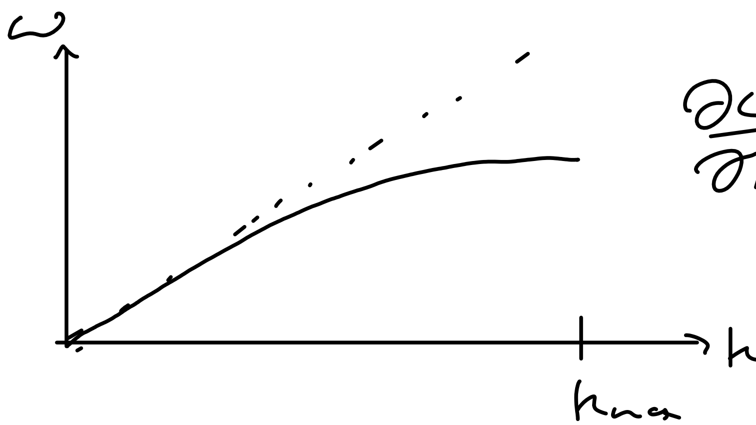
$$\frac{\partial p}{\partial t} = -\frac{\partial u}{\partial x} \qquad \frac{\partial u}{\partial x}_j = -\frac{u_{j+1/2} - u_{j-1/2}}{\Delta x}$$

$$-i\omega p_0 = -u_0 \left(\frac{e^{ik\Delta x/2} - e^{-ik\Delta x/2}}{\Delta x} \right)$$

$$-i\omega p_0 = -u_0\, 2i \sin(k\Delta x/2)$$

$$\boxed{\omega^2 = \frac{4 \sin^2\left(\frac{k\Delta x}{2}\right)}{\Delta x^2}}$$

$$k_{max} = \pi/\Delta x$$



$$\frac{\partial\omega}{\partial k} > 0$$