Two-fluid equations

Contents

- Review derivation from Boltzmann equation
- Collision operator moments
- Types of closure

Boltzmann equation

$$\frac{\partial f}{\partial t} + \underset{\sim}{v} \cdot \nabla f + \frac{\underset{\sim}{F}}{m} \cdot \frac{\partial f}{\partial \underset{\sim}{v}} = C_s(f) \qquad f_s(t, \underset{\sim}{x}, \underset{\sim}{v})$$

$$s = e, i, \ldots$$

$$C_s(f) = \sum_{s'} C_{s,s'}(f_s, f_{s'})$$

usually assumed bilinear

$$n(t, \underset{\sim}{x}) = \int f(t, \underset{\sim}{x}, \underset{\sim}{v}) \, d\underset{\sim}{v}$$

$$n\underset{\sim}{u}(t, x) = \int \underset{\sim}{v} f(t, \underset{\sim}{x}, \underset{\sim}{v}) \, d\underset{\sim}{v}$$

$$\vdots$$

$$\int \underset{\sim}{v}^n f(t, \underset{\sim}{x}, \underset{\sim}{v}) \, d\underset{\sim}{v}$$

Moments of the distribution function

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\underset{\sim}{u}) = 0 \qquad \int C(f) \, d\underset{\sim}{v} = 0$$

$$m\left[\frac{\partial n\underset{\sim}{u}}{\partial t} + \nabla \cdot (n\underset{\sim}{u}\underset{\sim}{u})\right] + \nabla \cdot \underset{=}{P} = \underbrace{q}_{\text{charge}} n (\underset{\sim}{E} + \underset{\sim}{u} \times \underset{\sim}{B}) + \underbrace{\underset{\sim}{R}}_{\text{friction}}$$

Pressure tensor

$$m \int C(f) \, \underset{\sim}{v} \, d\underset{\sim}{v}$$

$$\underline{\underline{P}} = \int m \underset{\sim}{w}\underset{\sim}{w} f(t,\underset{\sim}{x},\underline{v})\, dv$$

$$\underset{\sim}{w} = \underset{\sim}{v} - \underset{\sim}{u}$$

$$= \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$\frac{1}{3}\mathrm{Tr}[\underline{\underline{P}}] = p$$ scalar pressure

$$\begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix}$$ ideal MHD

$$\frac{3}{2}p = \int \frac{1}{2} m w^2 f(t,\underset{\sim}{x},\underline{v})\, d\underline{v}$$ internal energy

$$\underline{\underline{P}} = p\,\underline{\underline{\mathbb{I}}} + \underline{\underline{\Pi}}$$

$\underline{\underline{\Pi}}$: viscosity tensor

p: scalar pressure

$$R = \sum_{s'} m_s C_{s,s'}(f_s, f_{s'})\, \underset{\sim}{v}\, d\underline{v}$$

$$\int m_s C_{s,s'}(f_s, f_{s'})\, \underset{\sim}{v}\, dv$$

$$= -\int m_{s'} C_{s',s}(f_{s'}, f_s)\, \underset{\sim}{v}\, dv$$

conservation of momentum

$$\frac{3}{2}\frac{\partial p}{\partial t} + \nabla\cdot\left(\frac{3}{2}p\underset{\sim}{u}\right) + \underline{\underline{P}} : \nabla\underset{\sim}{u} + \nabla\cdot\underset{\sim}{q} = W_s$$

W_s: heat exchange with other species

$$P_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha}$$

$\underset{\sim}{q}$: heat flux

2 $$\underset{\sim}{q} = \int \frac{1}{2} m w^2 \underset{\sim}{w} f(t,\underset{\sim}{x},\underset{\sim}{v})\, d\underline{v}$$

$$W_s = \sum_{s'} W_{s,s'}(f_s, f_{s'})$$

$$W_s = \sum_{s'} \frac{1}{2} m w^2 C_{s,s'}(f_s, f_{s'})$$

Energy conservation : Internal + kinetic

$$W_{s,s'} + (\underset{\sim}{u}_s - \underset{\sim}{u}_{s'}) \cdot \underset{\sim}{R}_{s,s'} = -W_{s',s}$$

friction

Missing terms

- Viscosity tensor $\underline{\underline{\Pi}}$
- Heat flux $\underset{\sim}{q}$
- Collision operator: $\underset{\sim}{R}$, W