Generalised Ohm's Law

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Momentum equation:

electron velocity

$$m_e\left(\frac{\partial}{\partial t}(n\underset{\sim}{v_e}) + \nabla\cdot(n\underset{\sim}{v_e}\underset{\sim}{v_e})\right) = -en(\underset{\sim}{E}+\underset{\sim}{v_e}\times\underset{\sim}{B}) - \nabla\cdot\underset{\approx}{P_e} + \underset{\sim}{R_e}$$

pressure tensor

collisions (→ Resistivity)

$$\underset{\sim}{J} = en(\underset{\sim}{V_i} - \underset{\sim}{V_e}) \qquad\qquad \underset{\sim}{V_e} = \underset{\sim}{V_i} - \frac{\underset{\sim}{J}}{en}$$

fluid flow $\quad \underset{\sim}{u} \simeq \underset{\sim}{V_i}$

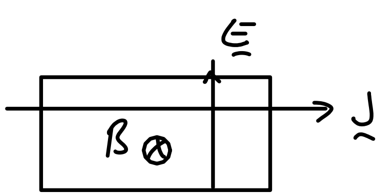
$$\underset{\sim}{V_e}\times\underset{\sim}{B} = \underset{\sim}{u}\times\underset{\sim}{B} - \frac{\underset{\sim}{J}\times\underset{\sim}{B}}{en}$$

Ideal MHD

$$\boxed{\underset{\sim}{E}+\underset{\sim}{u}\times\underset{\sim}{B}} = \frac{\underset{\sim}{J}\times\underset{\sim}{B}}{en} + \underbrace{\frac{\underset{\sim}{R_e}}{en}}_{\eta\underset{\sim}{J}} - \frac{\nabla\cdot\underset{\approx}{P_e}}{en} + \frac{m_e}{en}\left[\frac{\partial\underset{\sim}{J}}{\partial t} + \nabla\cdot\left(\underline{\underset{\sim}{u}\underset{\sim}{J}+\underset{\sim}{J}\underset{\sim}{u}} - \frac{\underset{\sim}{J}\underset{\sim}{J}}{en}\right)\right]$$

Hall term $\qquad$ Electron pressure $\qquad$ Electron inertia

Hall effect



Resistivity

$$\frac{\partial \underset{\sim}{B}}{\partial t} = -\nabla \times \underset{\sim}{E}$$

$$= \nabla \times (\underset{\sim}{u} \times \underset{\sim}{B} - \eta \underset{\sim}{J}) \qquad \underset{\sim}{J} = \frac{1}{\mu_0} \nabla \times \underset{\sim}{B}$$

$$= \frac{\eta}{\mu_0} \nabla^2 \underset{\sim}{B} \qquad \text{if } u \simeq 0,\ \eta \text{ constant}$$

Diffusion of the magnetic field

Important when

$$\frac{\eta}{\mu_0} \nabla^2 \underset{\sim}{B} \sim \nabla \times (\underset{\sim}{u} \times \underset{\sim}{B})$$

$$\nabla \sim \frac{1}{L}$$

$$\frac{\eta}{\mu_0} \frac{1}{L^{\cancel{2}}} \cancel{B} \sim \frac{1}{\cancel{L}} u \cancel{B}$$

Magnetic Reynolds number

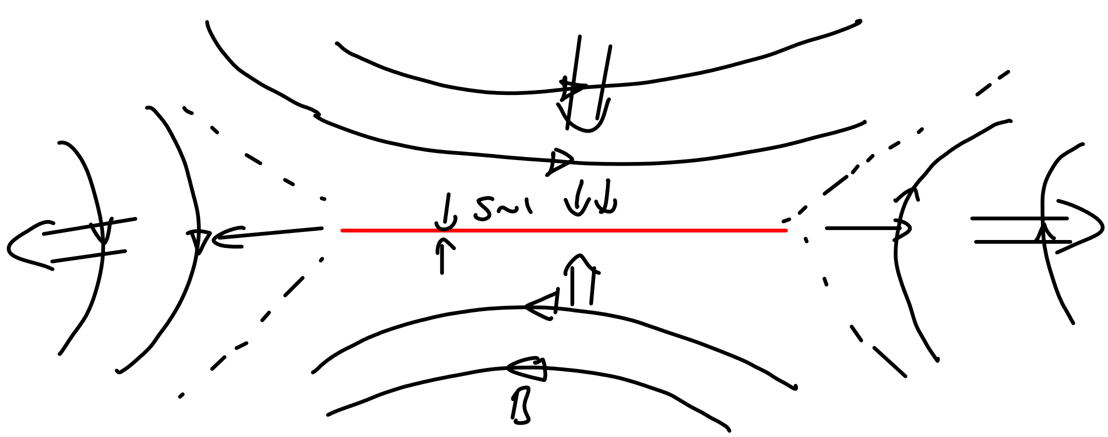
$$\underbrace{\frac{\mu_0 u L}{\eta}}_{\text{if } \uparrow \, \gg 1} \sim 1$$

then resistivity can be neglected

$u \sim V_A$ Alfven speed

$$\frac{\mu_0 V_A L}{\eta} = S \quad \text{Lundquist number}$$

$$\sim 10^6 - 10^7$$



Hall term

$$\frac{\underset{\sim}{J} \times \underset{\sim}{B}}{en} \sim \underset{\sim}{u} \times \underset{\sim}{B}$$

$$\underset{\sim}{J} = \frac{1}{\mu_0} \nabla \times B \qquad \nabla \sim \frac{1}{L}$$

$$\frac{B^{\cancel{2}}}{Len\mu_0} \sim u \cancel{B}$$

$$\frac{\delta_i}{L} \sim \frac{u}{V_A}$$

ion skin depth or inertial length

$$\delta_i = \sqrt{\frac{m_i}{\mu_0 e^2 n}}$$

$$V_A = \frac{B}{\sqrt{\mu_0 m_i n}}$$

on length scales $L \sim \delta_i$ the Hall term becomes important

$$\delta_i \sim 2\,\text{cm} \quad \text{if} \quad n \sim 10^{20}\,\text{m}^{-3}$$

Electron pressure term

$$\frac{\nabla \cdot \underline{\underline{P}}_e}{en} \qquad \nabla p = \underset{\sim}{J} \times B$$

$$P_e \simeq P_i$$

$\Rightarrow$ same scale as $\frac{\underset{\sim}{J} \times B}{en}$ Hall term

Electron mass

$$\frac{m_e}{en} \nabla \cdot (\underset{\sim}{J}\, \underset{\sim}{u}) \sim \underset{\sim}{u} \times \underset{\sim}{B}$$

$$\frac{m_e}{en}\frac{1}{L^2}\frac{\cancel{B}\,\alpha}{\mu_0} \sim \cancel{\partial_t B}$$

δ_e^2 electron inertiol length

$$\frac{\delta_e^2}{L^2} \sim 1 \qquad \delta_e \sim 0.5\,\mathrm{mm}$$

$$n \sim 10^{20}\,\mathrm{m}^{-3}$$