# Hall MHD

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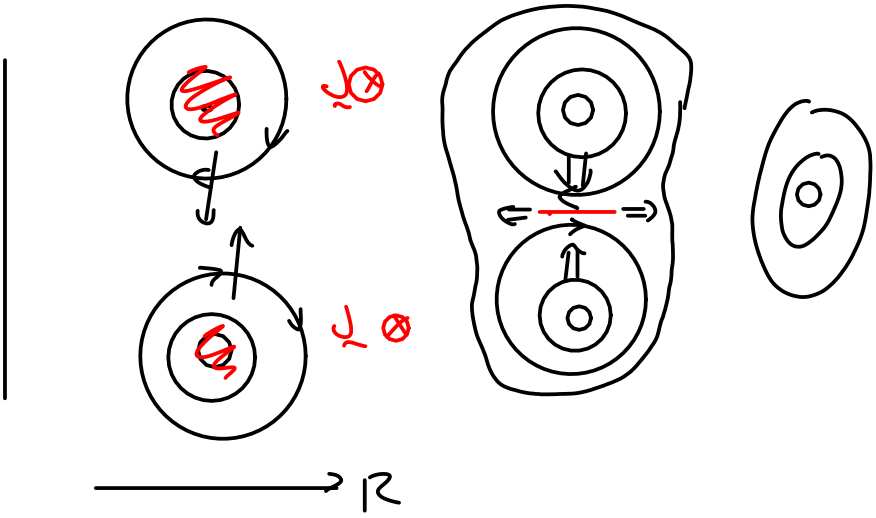
$$\underset{\sim}{E} + \underset{\sim}{u} \times \underset{\sim}{B} = \frac{1}{en}\left[\underset{\sim}{J} \times \underset{\sim}{B} - \nabla p_e\right]$$

causes magnetic field to be frozen into the electron fluid

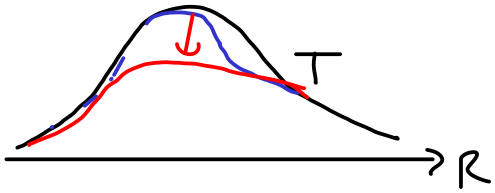
$$\underset{\sim}{E} + \underset{\sim}{v_e} \times \underset{\sim}{B} = -\frac{\nabla p_e}{en}$$

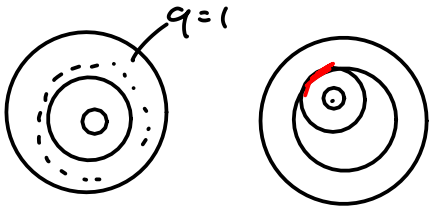
Important in magnetic reconnection

① Merging compression




② Sawtooth crash

Resistivity $\frac{\mu_0 V L}{\eta} = S \sim 1$

$$\eta \simeq 10^{-4} z \ln \Lambda T^{-3/2} \Omega m \quad [T \text{ in eV}]$$

$$\simeq 3 \times 10^{-8} \Omega m \quad @ \; T \sim 1 \text{ keV}$$

$B \sim 5T$
$n \sim 10^{20} m^{-3}$

$$V_A = \frac{B}{\sqrt{\mu_0 m_i n}} \sim 7.7 \times 10^6 \, m/s$$

$$L \simeq \frac{\eta}{\mu_0 V_A} \simeq \underline{\underline{3 \times 10^{-9} m}}$$

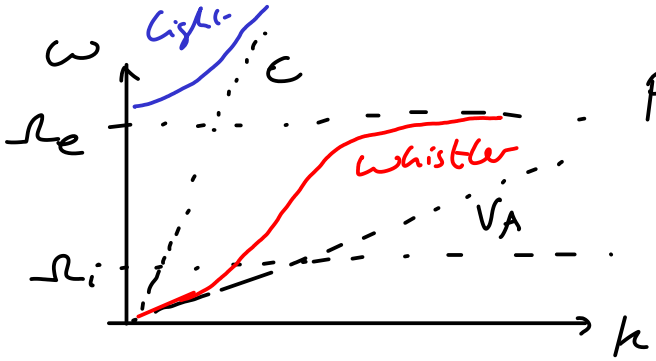
C.f. Hall term $\delta_i \sim L$ $\quad \delta_i \sim 2cm$

$n = 10^{20} m^{-3}$

## Dispersive waves

wave speeds depend on wavenumber $k$

e.g. whistler waves, kinetic Alfven waves




parallel

$k \parallel$ to $B$

right handed

$V_g = \frac{\partial \omega}{\partial k} \propto \sqrt{\omega}$ $\Rightarrow$ high frequency waves move faster

## Biermann battery

$$\underline{E} + \underline{u} \times \underline{B} = -\frac{\nabla p_e}{en}$$

Faraday's equation

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{u} \times \underline{B}) + \underbrace{\nabla \times \left( \frac{\nabla p_e}{en} \right)}_{-\frac{\nabla n \times \nabla p_e}{en^2}}$$

Can generate magnetic field, starting from a state with $B = 0$