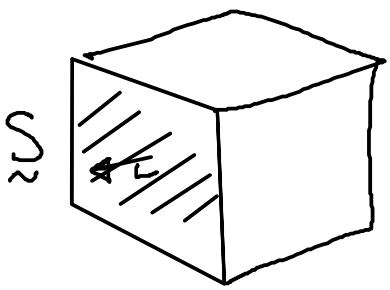
Fluid Equations

conservation
mass
momentum
energy



Volume V

Mass: $m = \int_V \rho(t, \underset{\sim}{x})\, dV$

$\rho(t, \underset{\sim}{x})$ mass density kg/m^3

flow through S

$$= \int_S \underbrace{\rho \underset{\sim}{u}}_{\text{kg/s/m}^2} \cdot d\underset{\sim}{S}$$

$$\frac{\partial m}{\partial t} = \frac{\partial}{\partial t}\int_V \rho\, dV = -\oint_S \rho \underset{\sim}{u} \cdot d\underset{\sim}{S} = -\int_V \nabla \cdot (\rho \underset{\sim}{u})\, dV$$

Volume $\longrightarrow 0$

$$\boxed{\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \underset{\sim}{u})}$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \underset{\sim}{u}) + S$$

source or sink

momentum $m\underset{\sim}{u} = \int_V \rho \underset{\sim}{u} \, dV$

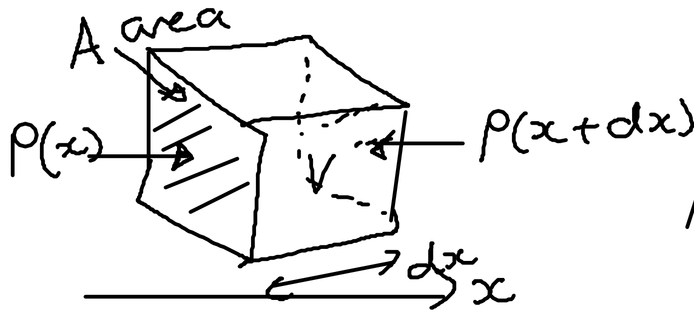
$\underset{\sim}{u} = (u_x, u_y, u_z)$

x direction: $m u_x$

$$\frac{\partial}{\partial t}\int_V \rho u_x \, dV = -\oint \rho u_x \underset{\sim}{u} \cdot d\underset{\sim}{S}$$

$$= -\int_V \nabla \cdot (\rho u_x \underset{\sim}{u}) \, dV + \text{Force}$$

$$\frac{\partial}{\partial t}(\rho u_x) = -\nabla \cdot (\rho u_x \underset{\sim}{u}) + F$$



$$\text{Force} = P(x)A - P(x+dx)A$$

$$F = \frac{\text{force}}{\text{volume}} = \frac{(P(x) - P(x+dx))\cancel{A}}{\cancel{A}\,dx}$$

volume $= A \times dx$

$$= -\frac{dP}{dx}$$

$$\frac{\partial}{\partial t}(\rho u_x) = -\nabla \cdot (\rho u_x \underset{\sim}{u}) - \frac{dP}{dx}$$

$$\boxed{\frac{\partial}{\partial t}(\rho \underset{\sim}{u}) = -\nabla \cdot (\rho \underset{\sim}{u} \underset{\sim}{u}) - \nabla P}$$

__Energy__ $E = e + \frac{1}{2}\rho u^2$

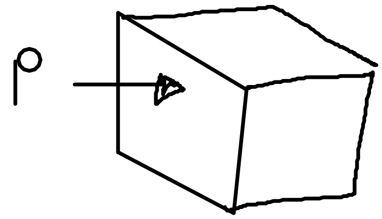
intend energy $e = \frac{1}{\gamma - 1} P = \frac{3}{2} P$

ratio of specific heats

$$\frac{\partial}{\partial t}\int E \, dV = -\oint E \underset{\sim}{u} \cdot d\underset{\sim}{S} + \text{Source of energy}$$

= work done

+ Heat



$$P \underset{\sim}{u} \cdot d\underset{\sim}{S} = J/s$$

$J/m^3 \; m/s \; m^2$

total work done $= \oint_S P \underset{\sim}{u} \cdot d\underset{\sim}{S}$

$$\frac{\partial}{\partial t}\int E \, dV = -\oint E \underset{\sim}{u} \cdot dS - \oint P \underset{\sim}{u} \cdot dS$$

$$\frac{\partial E}{\partial t} = -\nabla \cdot (E \underset{\sim}{u}) - \nabla \cdot (P \underset{\sim}{u}) \quad + \text{heat}$$

closure problem: Each equation depends on the next

heat flow $\to 0$ __adiabatic__

__Euler equations__

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underset{\sim}{u}) = 0 \qquad \text{mass density}$$

$$\frac{\partial}{\partial t}(\rho \underset{\sim}{u}) + \nabla \cdot (\rho \underset{\sim}{u} \underset{\sim}{u}) = -\nabla P \qquad \text{momentum}$$

$$\frac{\partial}{\partial t}E + \nabla \cdot \left((E + P) \underset{\sim}{u} \right) = 0 \qquad \text{energy}$$