Forms of the Euler equation

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \underline{u}) = 0$$

$$\frac{\partial(\rho\underline{u})}{\partial t} + \nabla\cdot(\rho\underline{u}\,\underline{u}) = -\nabla P$$

$$\frac{\partial E}{\partial t} + \nabla\cdot((E+P)\underline{u}) = 0 \qquad E = \frac{3}{2}P + \frac{1}{2}\rho u^2$$

$$\rho\frac{\partial \underline{u}}{\partial t} + \underline{u}\frac{\partial \rho}{\partial t} + \underline{u}\,\nabla\cdot(\rho\underline{u}) + \rho(\underline{u}\cdot\nabla)\underline{u} = -\nabla P$$

$$\underline{u}\left[\cancel{\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\underline{u})}\right] \to 0$$

$$\rho\left[\frac{\partial \underline{u}}{\partial t} + \underline{u}\cdot\nabla\underline{u}\right] = -\nabla P$$

primitive form of momentum equation

$$\frac{\partial E}{\partial t} + \nabla\cdot((E+P)\underline{u}) = 0 \qquad E = \frac{3}{2}P + \frac{1}{2}\rho u^2$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}P\right) + \frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2\right) + \nabla\cdot\left(\frac{3}{2}P\underline{u}\right) + \nabla\cdot\left(\frac{1}{2}\rho u^2\underline{u}\right) + \nabla\cdot(P\underline{u}) = 0$$

$$\frac{\partial}{\partial t}(\rho)\frac{1}{2}u^2 + \rho\frac{\partial}{\partial t}\left(\frac{1}{2}u^2\right)$$

$$\frac{1}{2}u^2\,\nabla\cdot(\rho\underline{u}) + \rho\underline{u}\cdot\nabla\left(\frac{1}{2}u^2\right)$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}P\right) + \frac{5}{2}\nabla\cdot(P\underline{u})$$

$$+ \frac{1}{2}u^2\left[\cancel{\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\underline{u})}\right]_{\to 0} + \rho\underbrace{\left[\frac{\partial}{\partial t}\left(\frac{1}{2}u^2\right) + \underline{u}\cdot\nabla\left(\frac{1}{2}u^2\right)\right]}_{=0}$$

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}u^2\right) = \frac{\partial}{\partial t}\left(\tfrac{1}{2}\underline{u}\cdot\underline{u}\right) = \underline{u}\cdot\frac{\partial \underline{u}}{\partial t}$$

$$\underline{u}\cdot\nabla\left(\tfrac{1}{2}u^2\right) = \underline{u}\cdot(\underline{u}\cdot\nabla)\underline{u}$$

$$\frac{3}{2}\frac{\partial P}{\partial t} + \frac{5}{2}\nabla\cdot(P\underline{u}) + \underbrace{\rho\left[\underline{u}\cdot\frac{\partial \underline{u}}{\partial t} + \underline{u}\cdot(\underline{u}\cdot\nabla)\underline{u}\right]}_{\rho\underline{u}\cdot\left[\frac{\partial \underline{u}}{\partial t} + \underline{u}\cdot\nabla\underline{u}\right] = -\underline{u}\cdot\nabla P} = 0$$

$$\frac{3}{2}\frac{\partial P}{\partial t} + \frac{5}{2}\nabla\cdot(P\underline{u}) - \underline{u}\cdot\nabla P = 0$$

$$\frac{5}{2}\nabla\cdot(P\underline{u}) = \frac{3}{2}\nabla\cdot(P\underline{u}) + \underline{u}\cdot\nabla P + P\nabla\cdot\underline{u}$$

$$\frac{3}{2}\left[\frac{\partial P}{\partial t} + \underbrace{\nabla\cdot(P\underline{u})}_{P\nabla\cdot\underline{u} + \underline{u}\cdot\nabla P}\right] + \underbrace{P\nabla\cdot\underline{u}} = 0$$

$$\frac{3}{2}\left[\frac{\partial P}{\partial t} + \underline{u}\cdot\nabla P\right] + \underbrace{\frac{3}{2}P\nabla\cdot u + P\nabla\cdot\underline{u}}_{\frac{5}{2}P\nabla\cdot u} = 0$$

$$\frac{\partial P}{\partial t} + \underline{u}\cdot\nabla P + \frac{5}{3}P\nabla\cdot\underline{u} = 0$$

$$\frac{\partial \rho}{\partial t} + \underline{u} \cdot \nabla \rho = -\rho \nabla \cdot \underline{u}$$

$$\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u} = -\frac{\nabla P}{\rho}$$

$$\frac{\partial P}{\partial t} + \underline{u} \cdot \nabla P = -\frac{5}{3} P \nabla \cdot \underline{u}$$

Lagrangian form

convective derivatives $\quad \frac{\partial}{\partial t} + \underline{u} \cdot \nabla = \frac{D}{Dt}$