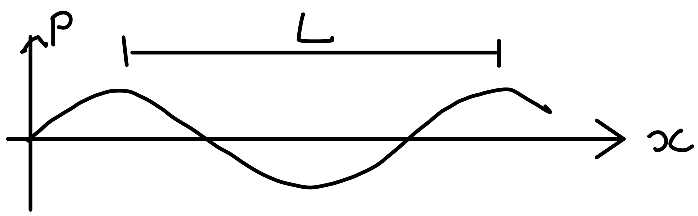
Validity of the fluid equations

Main concepts

- Time and length scale
- Particle distribution function
- Collisions
- Knudsen number

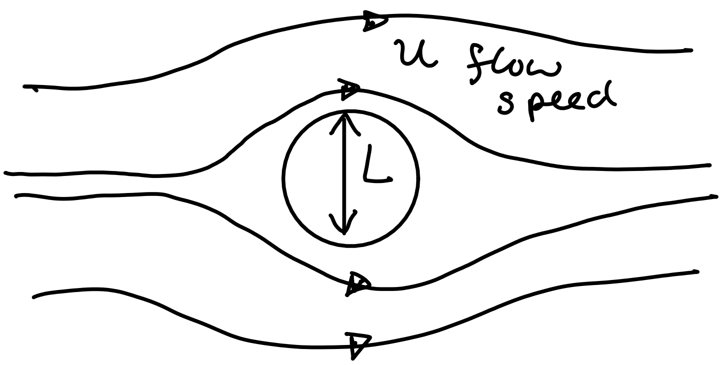
Typical time and length scales

waves



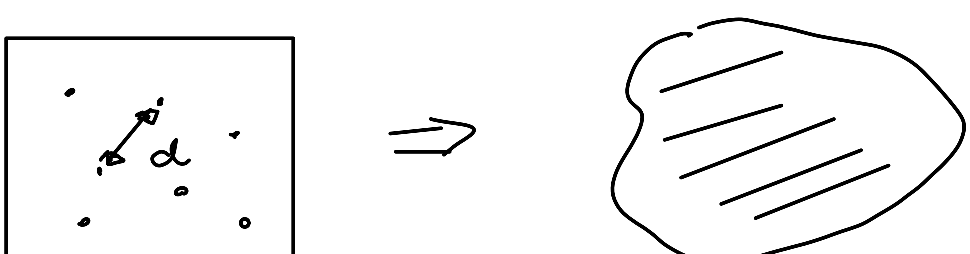
frequency ω timescale $T = \frac{2\pi}{\omega}$

flow



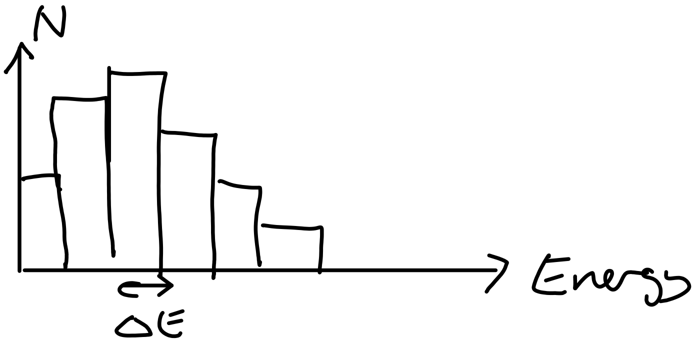
$T = \frac{L}{u}$

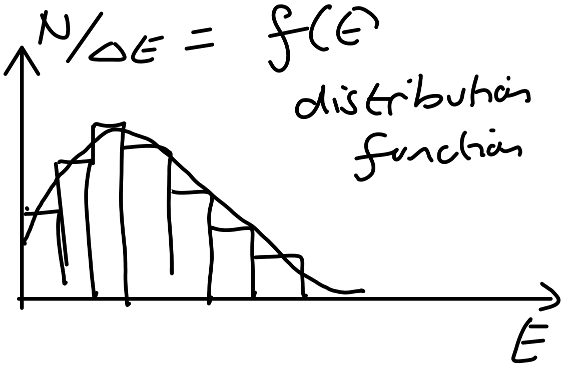
Microscopic



discrete continuous

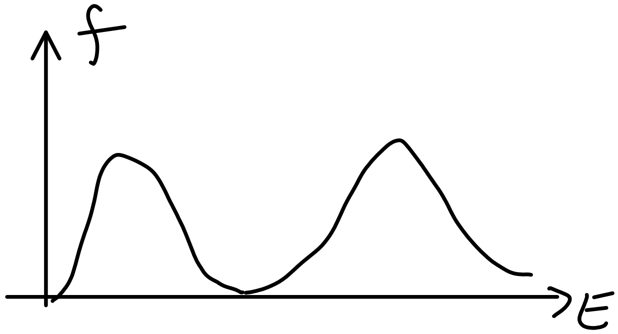
$d \ll L$ for fluid to be valid

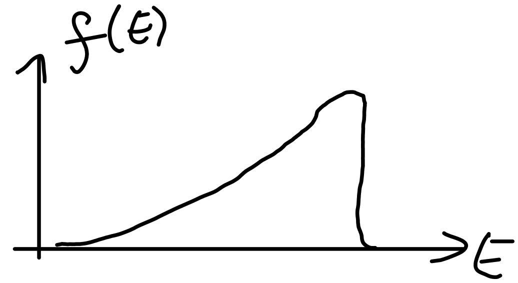




$$N = f(E)\Delta E$$

with particular energy

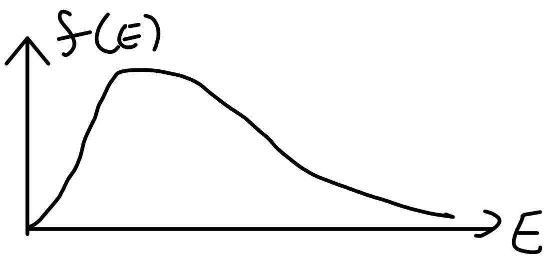




fluid approximation

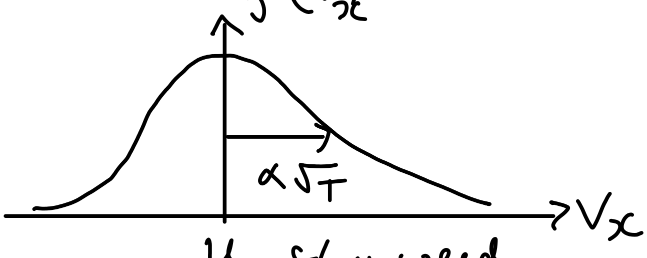
Describe $f(E)$ using only $\rho, \underset{\sim}{u}, P$

Maxwell - Boltzmann



$$f(E) = 2\sqrt{\frac{E}{\pi}}\left(\frac{1}{kT}\right)^{3/2} e^{-E/kT}$$

Local thermodynamic equilibrium



$$f(V_x) = \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-\frac{m}{kT}(V_x - u)^2}$$

3 parameters

$\rho, \underset{\sim}{u}, P \rightarrow T$

M–B is the solution when there are many collisions

Require

1) Collisions occur much more frequently than ω
(shorter time than T)

$$\nu_c \gg \omega$$

e.g. air $\nu_c \sim 10^{10}$ Hz

2) Distance between collision

$$\lambda_{mfp} \ll L$$

where L is length scale of interest

<u>Knudsen number</u> $\quad Kn = \dfrac{\lambda_{mfp}}{L} \ll 1$

e.g. air $\lambda_{mfp} \sim 68\,\text{nm}$