From kinetic theory to fluids

Concepts

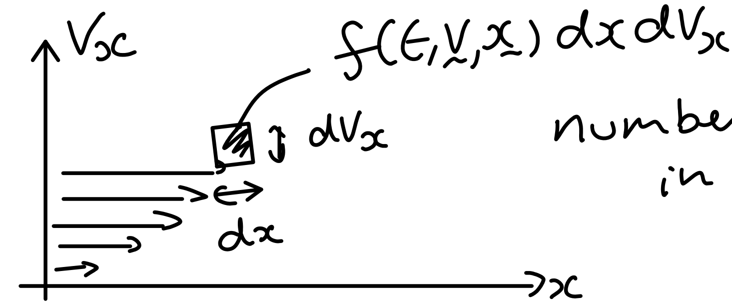
- Kinetic (Boltzmann) equation
- Moments of the kinetic equation
- The closure problem

Kinetic equation

Distribution function $f(t, \underset{\sim}{v}, \underset{\sim}{x})$

7D function

coordinate



number of particles in small region

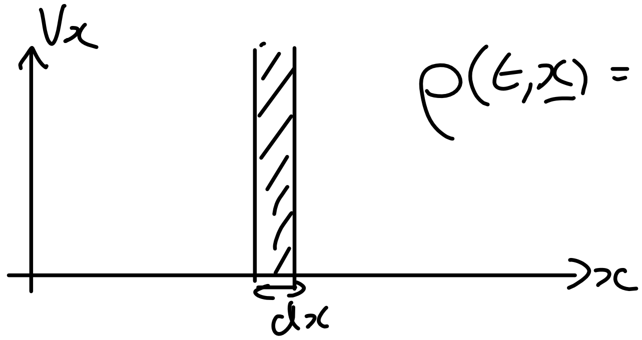
Boltzmann

$$\frac{\partial f}{\partial t} + \underset{\sim}{v} \cdot \nabla f + \frac{\underset{\sim}{F}}{m_\alpha} \cdot \frac{\partial}{\partial \underset{\sim}{v}} f = \underbrace{C(f)}_{\text{collision}}$$

Liouville equations
BBGKY

fluids $\rho(t, \underset{\sim}{x})$, $\underset{\sim}{u}(t, \underset{\sim}{x})$, $p(t, \underset{\sim}{x})$

Density



$$\rho(t,\underline{x}) = m_a \int f(t,\underline{v},\underline{x})\, d\underline{v}$$

Moments $\rightarrow$ 0^{th} moment

$$m_a \int \frac{\partial f}{\partial t} \underbrace{d\underline{v}}_{dv_x dv_y dv_z} + m_a \underbrace{\int \underset{\sim}{v}\cdot\nabla f\, d\underline{v}}_{\nabla\cdot\left(\int \underset{\sim}{v} f\, dv\right)} = m_a \int C(f)\, d\underline{v}$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot \underbrace{m_a\int \underset{\sim}{v} f\, d\underline{v}}_{\rho \underset{\sim}{u}} = m_a \int C\, d\underline{v}$$

$$\int v_x \frac{\partial f}{\partial x} dv_x = \frac{\partial}{\partial x}\int v_x f\, dv_x$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\underline{u}) = m_a \underbrace{\int C(f)\, dv}_{=0 \text{ if } f \text{ Maxwellian}}$$

$$m_a \int \underset{\sim}{v} \frac{\partial f}{\partial t} d\underline{v} + m_a \int \underset{\sim}{v}\, \underline{v}\cdot\nabla f\, d\underline{v} = m_a \int \underline{v}\, C(f)\, d\underline{v}$$

$$\int v^n \frac{\partial f}{\partial t} d\underline{v} \quad n^{th} \text{ moment}$$

$$\frac{\partial}{\partial t}(\rho\underline{u}) + \nabla\cdot\left[\underline{m_a \int \underset{\sim}{v}\underset{\sim}{v} f\, d\underline{v}}\right] = m_a \int \underline{v}\, C(f)\, d\underline{v}$$

$m_a \int \underline{v}\,\underline{v}\, f \, d\underline{v}$ flow of momentum in lab frame

write in terms of relative velocity

$$\underline{\omega} = \underline{v} - \underline{u}$$

$\underline{\omega}$: relative velocity

$\underline{u}$: fluid velocity

$$m_a \int \underline{v}\,\underline{v}\, f\, d\underline{v} = m_a \int (\underline{\omega} + \underline{u})(\underline{\omega} + \underline{u})\, f\, d\underline{v}$$

$$= m_a \int \underline{\omega}\,\underline{\omega}\, f\, d\underline{v}$$

$\underline{\underline{P}}$ pressure tensor

$$+ m_a \int \underline{\omega}\,\underline{u}\, f\, d\underline{v} + m_a \int \underline{u}\,\underline{\omega}\, f\, d\underline{v}$$

$\underline{u}(t, \underline{x})$

$$\underline{u} \int \underline{\omega}\, f\, d\underline{v} = 0$$

$$+ m_a \int \underline{u}\,\underline{u}\, f\, d\underline{v}$$

$$\underline{u}\,\underline{u} \int f\, d\underline{v}$$

$$= \underline{u}\,\underline{u}\, \frac{\rho}{\cancel{m_a}}$$

$$\frac{\partial}{\partial t}(\rho \underline{u}) + \nabla \cdot (\underline{u}\,\underline{u}\,\rho) + \nabla \cdot \underline{\underline{P}} = m_a \int \underline{v}\, C(f)\, d\underline{v}$$

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if Maxwellian $\nabla \cdot \underline{\underline{P}} \Rightarrow \nabla p$

$$m_a \int \underline{v}\, C(f)\, d\underline{v} \Rightarrow \text{viscosity}$$

① If Maxwellian, no viscosity

$\rightarrow$ Recover Euler

② Closure problem

$\int \underline{v}^n f \, d\underline{v}$ depends on $\int \underline{v}^{n+1} f \, d\underline{v}$

truncate, Chapman-Enskog