Whistler waves

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$$\Pi_e^2 = \frac{ne^2}{\varepsilon_0 m_e}$$

$$N_\parallel^2 = \left(\frac{ck_\parallel}{\omega}\right)^2 \simeq 1 - \frac{\Pi_e^2}{(\omega+\Omega_e)(\omega+\Omega_i)}$$

$$\Omega_e = \frac{-eB}{m_e} < 0$$

$$\Omega_i = \frac{zeB}{m_i}$$

① Low freq $\omega << \Omega_i \longrightarrow$ shear Alfven

$$N_\parallel^2 \simeq 1 - \frac{\Pi_e^2}{\Omega_e \Omega_i} \simeq 1 - \frac{e^2 n}{\varepsilon_0 m_e}\left(\frac{m_e}{-eB}\right)\left(\frac{m_i}{eB}\right)$$

$$\mu_0 \varepsilon_0 = \frac{1}{c^2}$$

$$\frac{c^2 k_\parallel^2}{\omega^2} \simeq 1 + \frac{n m_i \mu_0}{B^2} c^2 \simeq 1 + c^2/V_A^2$$

$$\frac{k_\parallel^2}{\omega^2} = \frac{1}{c^2} + \frac{1}{V_A^2}$$ Alfven wave

② Intermediate $\Omega_i << \omega << \Omega_e$

$$N_\parallel^2 = 1 - \frac{\Pi_e^2}{\underbrace{(\omega+\Omega_e)}_{\sim \Omega_e}\underbrace{(\omega+\Omega_i)}_{\omega}}$$

$$N_\parallel^2 \simeq \frac{-\Pi_e^2}{\Omega_e \omega} \qquad (\Pi_e \text{ large})$$

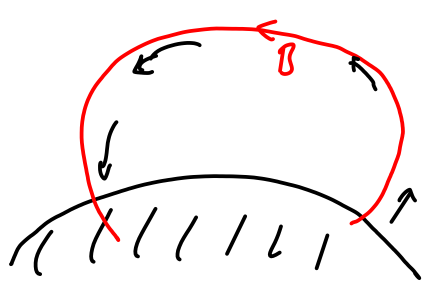
$$\boxed{c^2 k_\parallel^2 \simeq \frac{-\Pi_e^2}{\Omega_e}\omega}$$

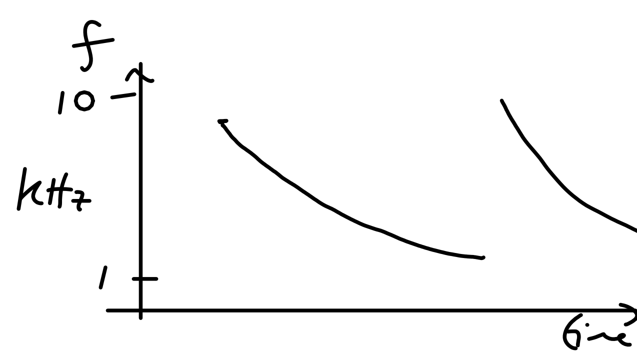
Group velocity $\frac{\partial \omega}{\partial k_\parallel}$

$$2c^2 k_\parallel = \frac{-\Pi_e^2}{\Omega_e}\frac{\partial \omega}{\partial k_\parallel}$$

$$2c\left(\frac{\Pi_e \sqrt{\omega}}{\sqrt{-\Omega_e}}\right) \simeq \frac{-\Pi_e^2}{\Omega_e}\frac{\partial \omega}{\partial k_\parallel}$$

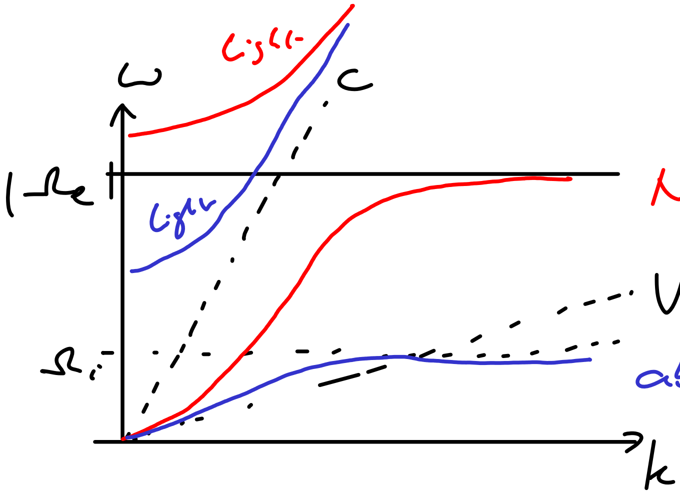
$$\frac{\partial \omega}{\partial k_\parallel} \simeq 2c\frac{\sqrt{\omega|\Omega_e|}}{\Pi_e} \qquad \propto \sqrt{\omega}$$





Time delay $\propto \int \frac{\Pi_e}{\sqrt{\omega/\Omega_e}}\, dl$

Most sensitive to plasma density at top of loop.



$N_\parallel^2 \to \infty$ absorbed R electron whistlers

absorbed at ion cyclotron frequency L waves

$\omega \sim \Omega_e$ R wave

$$N_\parallel^2 \simeq -\frac{\Pi_e^2}{(\omega+\Omega_e)\,\omega} \qquad c^2 k_\parallel^2 = -\frac{\Pi_e^2\,\omega}{\omega+\Omega_e}$$

$$2c^2 k_\parallel(\omega+\Omega_e) + c^2 k_\parallel^2 \frac{\partial\omega}{\partial k_\parallel} \simeq -\Pi_e^2 \frac{\partial \omega}{\partial k_\parallel}$$

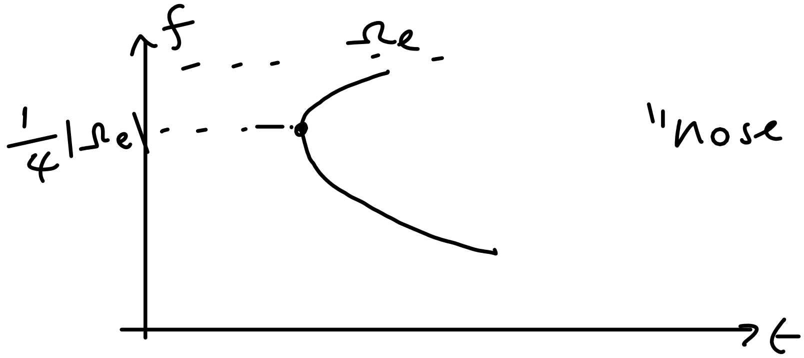
$$2c^2 \sqrt{\frac{-\Pi_e^2\,\omega}{\omega+\Omega_e}}\,(\omega+\Omega_e) \simeq -\left(c^2 k_\parallel^2 + \Pi_e^2\right)\frac{\partial\omega}{\partial k_\parallel}$$

$$\frac{-\Pi_e^2\,\omega + \Pi_e^2(\omega+\Omega_e)}{\omega+\Omega_e} = \frac{\Pi_e^2\,\Omega_e}{\omega+\Omega_e}$$

$$2c^2\sqrt{\omega}\,(\omega+\Omega_e)^{3/2} \sim -\Pi_e\,\Omega_e \frac{\partial\omega}{\partial k_\parallel}$$

$\frac{\partial\omega}{\partial k_\parallel} \to 0$ as $\omega \to \Omega_e$ $\Rightarrow$ Max. velocity

Max. group velocity is at $\omega = \frac{1}{4}|\Omega_e|$



"nose whistles"