Bohm sheath

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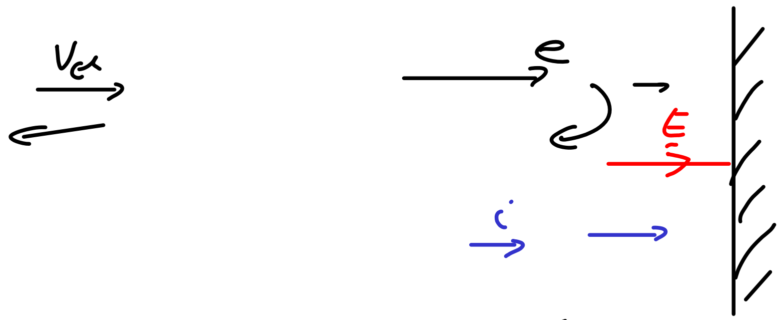

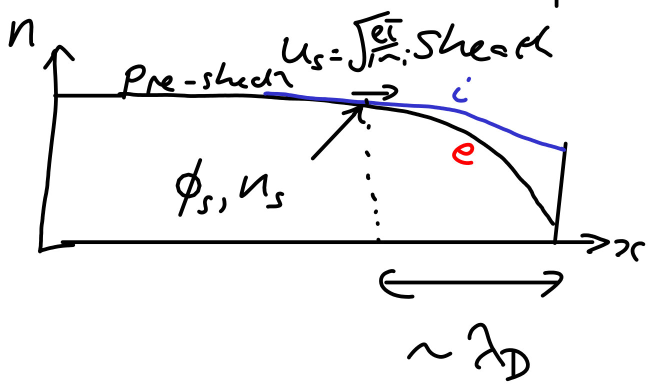

$$\lambda_D = \sqrt{\frac{\varepsilon_0 T_e}{e n_e}}$$

$\sim \mu m$

$$\nabla \cdot \underset{\sim}{E} = \frac{\rho}{\varepsilon_0} = \frac{e(n_i - n_e)}{\varepsilon_0}$$

$$\underset{\sim}{E} = -\nabla\phi \qquad -\frac{d^2\phi}{dx^2} = \frac{e(n_i - n_e)}{\varepsilon_0}$$

Electrons

Unmagnetised

Steady state

$$\underset{\sim}{E} + \cancel{\underset{\sim}{v_e} \times \underset{\sim}{B}} = -\frac{\nabla p_e}{en} + \cancel{\frac{m_e}{en}\frac{dv}{dt}}$$

Small m_e

$T_e \sim \text{constant} \qquad p_e = e n_e T_e$

$$-\frac{d\phi}{dx} = -\frac{T_e}{n}\frac{dn_e}{dx}$$

$$\int \frac{d\phi}{dx}\,dx = \int \frac{T_e}{n}\,dn = T_e \ln(n)$$

$$\boxed{n_e = n_s e^{(\phi - \phi_s)/T_e}}$$

Boltzmann relation

Ion density

steady state, collisionless, unmagnetised

$$m_i n\left(\cancel{\frac{\partial u_i}{\partial t}} + u_i \cdot \nabla u_i\right) = -\cancel{\nabla p_i} - \cancel{\nabla \cdot \Pi} + en(E + \cancel{u_i \times B})$$

cold ions
thermal speed << drift speed

$$m_i \cancel{n_i} u_i \underbrace{\frac{du_i}{dx}}_{\frac{1}{2}\frac{d}{dx}(u_i^2)} = -e\cancel{n_i}\frac{d\phi}{dx}$$

$$\frac{d}{dx}\underbrace{\left(\frac{1}{2}m_i u_i^2 + e\phi\right)}_{\text{constant}} = 0$$

$$\frac{1}{2}m_i u_s^2 + e\phi_s = \frac{1}{2}m_i u_i^2 + e\phi$$

$$u_i = u_s\sqrt{1 - \frac{e(\phi-\phi_s)2}{u_s^2 m_i}}$$

$$\cancel{\frac{\partial n_i}{\partial t}} = -\nabla\cdot(n_i u_i) \qquad \frac{d}{dx}(n_i u_i) = 0$$

0

$$\boxed{n_i = n_s\left(1 - \frac{2e(\phi-\phi_s)}{u_s^2 m_i}\right)^{-1/2}}$$

Boltzmann

$$-\frac{d^2\phi}{dx^2} = \frac{e(n_i - n_e)}{\varepsilon_0}$$

$$= en_s\left[\left(1 - \frac{2e}{u_s^2 m_i}(\phi-\phi_s)\right)^{-1/2} - e^{(\phi-\phi_s)/Te}\right]$$

normalise $\quad \Phi = -\frac{(\phi-\phi_s)}{Te} \qquad y = \frac{\sqrt{2}\,x}{\lambda_D}$

$$k = \frac{m_i u_i^2}{2eTe}$$

$$2\frac{d^2\Phi}{dy^2} = e^{-\Phi} + \left(1 + \frac{\Phi}{k}\right)^{-1/2}$$

$\times \frac{d\Phi}{dy}$, integrate in y

$$\left(\frac{d\Phi}{dy}\right)^2 = e^{-\Phi} - 1 + 2k\left[\underbrace{\left(1+\frac{\Phi}{k}\right)^{1/2}}_{} - 1\right]$$

$\geqslant 0$ for all y

expand for small Φ (near sheath entrance)

$$\left(\frac{d\Phi}{dy}\right)^2 \simeq \overbrace{\left(\cancel{1} - \cancel{\Phi} + \tfrac{1}{2}\Phi^2 - \cdots\right)} - \cancel{1}$$

$$+ 2k\left[\overbrace{\left(\cancel{1} + \cancel{\tfrac{1}{2}\frac{\Phi}{k}} - \frac{1}{8}\frac{\Phi^2}{k^2} + \cdots\right)} - \cancel{1}\right]$$

$$\simeq \frac{1}{2}\Phi^2\left(1 - \frac{1}{2k}\right) + O(\Phi^3) + \cdots$$

$$\geqslant 0 \qquad \Rightarrow \qquad \underline{\underline{k \geqslant \frac{1}{2}}}$$

$$\frac{m_i u_i^2}{2eT_e} \geqslant \frac{1}{2} \qquad \underline{\underline{u_s \geqslant \sqrt{\frac{eT_e}{m_i}}}}$$

Bohm criterion at sheath