Chodura Sheath

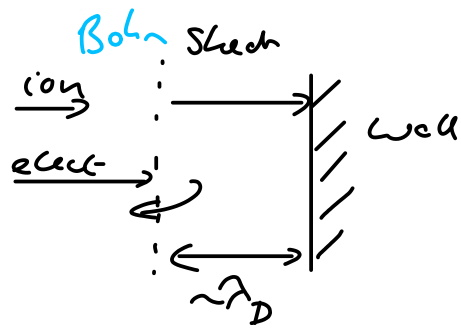
Contents

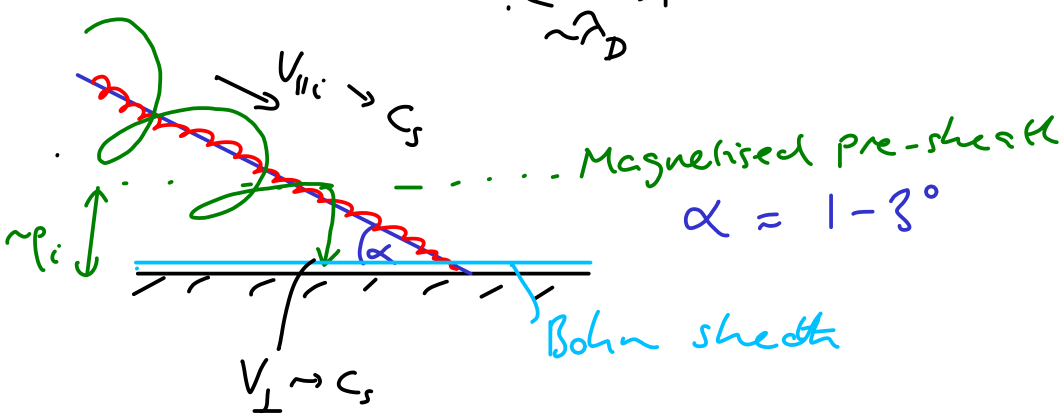
- Magnetised pre-sheath
- Bohm-Chodura-Riemann analysis
- Result: Parallel ion flow goes to sound speed

References

- P. Stangeby PoP 2, 702 (1995)
- K-U Riemann PoP 1, 552 (1994)
- R. Chodura, Phys Fluids 25, 1628 (1982)

sheath





- Quasineutral $n_e \simeq n_i$
- $\rho_i \gg \lambda_D \Rightarrow$ ignore ion orbit in Debye sheath
- Boundary condition on parallel ion flow and heat flux is ~same as Bohm
- Critical small angle $\alpha \sim \dfrac{V_{thi}}{V_{the}} \sim \sqrt{\dfrac{m_e}{m_i}} \sim \dfrac{1}{60}$

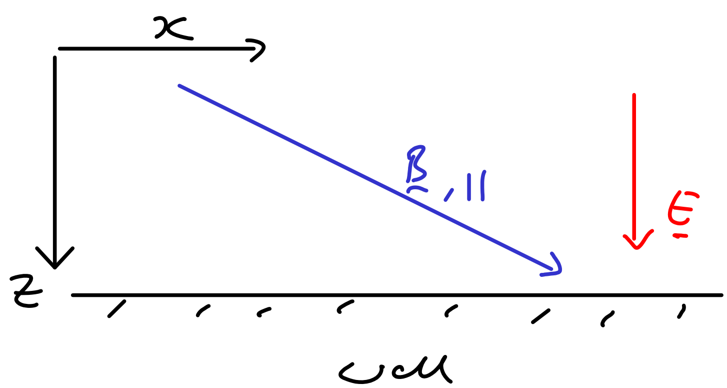
$\Rightarrow$ Significant change in sheath
(ions reach wall first)

Analysis

Quasineutral $n_i = n_e$

Boltzmann response for electrons $n_e = n_0 e^{\phi/T_e}$

$$\nabla\cdot(n_i \underline{u}) = 0 \qquad m_i \underline{u}\cdot\nabla\underline{u} = e(\underset{\uparrow\ -\nabla\phi}{\underline{E}} + \underline{u}\times\underline{B}) - \frac{1}{n}\nabla p_i$$



$\underline{E}\times\underline{B}\ \otimes$

Only variation in z

z component:
$$m_i u_z \frac{d}{dz} u_z = e(\underset{\uparrow\ -\frac{d\phi}{dz}}{E_z} - B_x u_{E\times B}) - \underbrace{\frac{1}{n}\frac{d\overset{eT_i n}{\downarrow}{p_i}}{dz}}_{eT_i\frac{d}{dz}\ln n}$$

$n = n_e = n_i$

$$\ln n = \ln n_0 + \frac{\phi}{T_e}$$

$$m_i u_z \frac{d}{dz} u_z = -eB_x u_{E\times B} - e\frac{d}{dz}\left(\phi + T_i\frac{\phi}{T_e}\right) \qquad (T_i, T_e \text{ const})$$

$$= -eB_x u_{E\times B} - e\left(1 + \frac{T_i}{T_e}\right)\frac{d\phi}{dz}\ *$$

ϕ ?

$$\nabla\cdot(n\underline{u}) = 0$$

$$\frac{d}{dz}(n u_z) = 0 \quad \Rightarrow \quad \frac{d}{dz}\left(n_0 e^{\phi/T_e} u_z\right) = 0$$

$$n_0 e^{\phi/T_e} u_z = \text{const}$$
$$= n_0 c_s$$

Define $\phi = 0$ at entrance to Debye sheath
$n = n_0 \quad u_z = c_s$

$$\phi^{*} = -T_e \ln\left(\frac{u_z}{c_s}\right)$$

$$\Rightarrow \quad u_z \frac{d}{dz} u_z = \frac{-eB_x}{m_i} u_{E\times B} + \frac{c_s^2}{u_z}\frac{du_z}{dz}$$

$$\boxed{\left(u_z - \frac{c_s^2}{u_z}\right)\frac{du_z}{dz} = \frac{-eB_x}{m_i} u_{E\times B}}$$

<u>E×B component</u>

$$u_z \frac{d}{dz} u_{E\times B} = \frac{eB_x}{m_i} u_z - \frac{eB_z}{m_i} u_x$$

Energy (combine $u_z, u_x, u_{E\times B}$)

flow into magnetised presheath

$$u_z^2 + u_x^2 + u_{E\times B}^2 = 2 c_s^2 \ln\left(\frac{u_z}{u_{z0}}\right) + V_{\parallel}^{MPS}$$

$$\boxed{\left(\frac{c_s^2 - u_z^2}{c_s u_z}\right)^2 \left(\rho_i \frac{du_z}{dz}\right)^2 = f(u_z)} = (V_{\parallel}^{MPS})^2 + 2 c_s^2 \ln\left(\frac{u_z}{u_{z0}}\right) - u_z^2 - \left[\frac{V_{\parallel}^{MPS} + c_s^2/V_{\parallel}^{MPS}}{\cos\alpha} - \delta\left(u_z + \frac{c_s^2}{u_z}\right)\right]^2$$

c_s/Ω_i

$\Omega_i = eB/m_i$

$\delta = \tan\alpha = B_z/B_x$

$$\left(\frac{c_s^2 - u_z^2}{c_s u_z}\right) f^{-1/2}(u_z)\, \rho_i \frac{du_z}{dz} = 1$$

Integrate by separation of variables

$$\frac{z}{\rho_i} = -\int_{u_z}^{c_s \leftarrow \text{Debye sheath (Bohm } u_z = c_s)} \left(\frac{c_s^2 - u_z}{c_s u_z}\right) f(u_z)^{-1/2} \, du_z$$

Taylor expand $u_z \simeq u_{z0}$

at MPSE $u_z = u_{z0}$

$$f(u_{z0}) = 0 \qquad f'(u_{z0}) = 0$$

$$f''(u_{z0}) = \frac{2\delta^2}{u_{z0}}\left(\frac{c_s^2}{u_{z0}^2} - 1\right)\left(V_{\parallel}^{MPS} - c_s^2\right)$$

≥ 0 if $f^{1/2}(u_z)$ is real

$$\Rightarrow \underline{\underline{V_{\parallel}^{MPS} \geq c_s}}$$ Chodura condition