Polarisation drift

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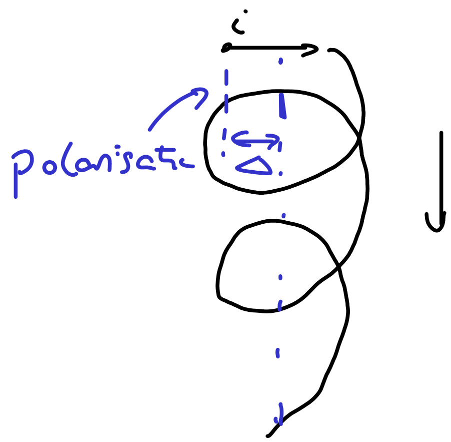
$\underset{\sim}{B} \odot \qquad \circ^{i} \qquad \underset{\sim}{v} = 0$ at $t = 0$

$$\Rightarrow \underset{\sim}{E} \longrightarrow \underset{\sim}{x}$$



$$v_{drift} = \frac{\underset{\sim}{E} \times \underset{\sim}{B}}{B^2}$$

Single particle

$$m \frac{d\underset{\sim}{v}}{dt} = q(\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B})$$

$$\times \frac{\underset{\sim}{B}}{B^2}$$

$$-\underset{\sim}{v} + \frac{(\underset{\sim}{v} \cdot \underset{\sim}{B})\underset{\sim}{B}}{B^2} = -\underset{\sim}{v}_{\perp}$$

$$\frac{m}{qB^2} \frac{d\underset{\sim}{v}}{dt} \times B = \frac{\underset{\sim}{E} \times \underset{\sim}{B}}{B^2} + \overbrace{\frac{(\underset{\sim}{v} \times \underset{\sim}{B}) \times \underset{\sim}{B}}{B^2}}$$

$$\underset{\sim}{V}_\perp = \frac{\underset{\sim}{E}\times\underset{\sim}{B}}{B^2} - \underbrace{\frac{m}{qB^2}\frac{d\underset{\sim}{v}}{dt}}_{\sim \omega/\Omega_i}\times\underset{\sim}{B}$$

$\frac{d}{dt} \sim \omega$ frequency

$\frac{qB}{m} \simeq \Omega_i$ cyclotron frequency

$$V_\perp^{(1)} \simeq \frac{E\times B}{B^2} + O\left(\frac{\omega}{\Omega_i}\right)$$

$$V_\perp^{(2)} \simeq \frac{E\times B}{B^2} + \frac{m}{qB^2}\frac{d}{dt}\left(\frac{E\times B}{B^2}\right)\times\underset{\sim}{B} + O\left(\omega^2/\Omega_i^2\right)$$

$\underset{\sim}{B} \simeq$ const $\quad \frac{(E\times B)\times B}{B^2} = -\underset{\sim}{E}_\perp$

$$V_\perp^{(2)} = \frac{E\times B}{B^2} + \underbrace{\frac{m}{qB^2}\frac{d}{dt}\underset{\sim}{E}_\perp}_{\text{polarisation drift}}$$

$$\Delta x = \int V_\perp \, dt = \int \frac{m}{qB^2}\frac{d}{dt}E_x \, dt$$

$$= \frac{m}{qB^2}\Delta E_x$$

usually ignore for electrons
because m_e small.

Derivation from fluid equations

$$\rho\left[\frac{\partial \underset{\sim}{u}}{\partial t} + \underset{\sim}{u}\cdot\nabla \underset{\sim}{u}\right] = -\nabla p + \underset{\sim}{J}\times \underset{\sim}{B}$$

$$\frac{d\underset{\sim}{E}}{dt}$$

Curl $\rightarrow$ $\nabla\times\nabla p \rightarrow 0$

leads to a vorticity equation

Constant $\frac{\partial}{\partial t} \to 0$

$$\frac{\underset{\sim}{b}}{qB}\times\left[\rho\frac{\partial u}{\partial t}\right] = \underbrace{\cancel{\left(\nabla\times\frac{\underset{\sim}{b}}{B}\right)}}_{\text{small}}\cdot\frac{\rho}{q}\frac{\partial u}{\partial t} - \nabla\cdot\left(\frac{\rho\,\underset{\sim}{b}}{qB}\times\frac{\partial u}{\partial t}\right)$$

Ohm's law $\quad \underset{\sim}{E} + \underset{\sim}{u}\times\underset{\sim}{B} = 0$

$$\underset{\sim}{E}\times\underset{\sim}{B} + \underbrace{(\underset{\sim}{u}\times\underset{\sim}{B})\times\underset{\sim}{B}}_{-\underset{\sim}{u}_\perp B^2} = 0$$

$$\Rightarrow \quad \underset{\sim}{u}_\perp = \frac{\underset{\sim}{E}\times\underset{\sim}{B}}{B^2}$$

$$\frac{\underset{\sim}{b}}{qB}\times\left[\rho\frac{\partial u}{\partial t}\right] \approx -\nabla\cdot\left\{\underbrace{\frac{\rho}{qB^2}\frac{\partial}{\partial t}\underset{\sim}{E}_\perp}_{\text{polarisation drift}}\right\}$$