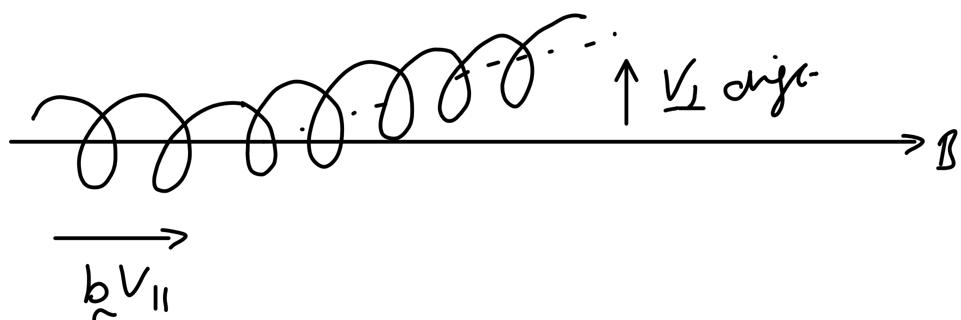
Drift-ordered equations

Contents

- Simple derivation of reduced equations from particle drifts
- Vorticity equation



$$\underset{\sim}{V} = \underset{\sim}{b} V_\parallel + \frac{\underset{\sim}{E} \times \underset{\sim}{B}}{B^2} + \underbrace{\frac{m}{qB^2} \frac{d\underset{\sim}{E}_\perp}{dt}}_{\text{polarisation}} + \underbrace{\frac{T}{qB} \frac{\underset{\sim}{B} \times \nabla B}{B^2}}_{\text{Magnetic drift}}$$

ions, electrons

$T_i = 0$ cold ion

$m_i \gg m_e$

$\Rightarrow$ neglect electron polarisation

Conservation

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \underset{\sim}{v}) = 0 \qquad n = n_e = n_i$$

$$\nabla \cdot \underset{\sim}{J} = 0 \qquad \text{quasineutrality}$$

Density

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (\underset{\sim}{b} V_{\parallel e} n_e) + \underbrace{\nabla \cdot \left(\frac{\underset{\sim}{E} \times \underset{\sim}{B}}{B^2} n_e \right)} + \nabla \cdot \left(\frac{T_e}{B} \frac{\underset{\sim}{B} \times \nabla B}{B^2} n_e \right) = 0$$

$$\frac{\underset{\sim}{E} \times \underset{\sim}{B}}{B^2} = \frac{\underset{\sim}{b} \times \nabla \phi}{B^2} \qquad \frac{\partial \underset{\sim}{A}}{\partial t} \rightarrow 0 \quad \text{Electrostatic}$$

$$\nabla\cdot\left(\frac{\underset{\sim}{E}\times\underset{\sim}{B}}{B^2}\, n_e\right) = \frac{1}{B}\underset{\sim}{b}\times\nabla\phi\cdot\nabla n_e + n_e\,\underbrace{\nabla\cdot\left(\frac{\underset{\sim}{b}\times\nabla\phi}{B}\right)}$$

$$\nabla\times\left(\phi\,\frac{\underset{\sim}{b}}{B}\right) = \phi\,\nabla\times\left(\frac{b}{B}\right) + \nabla\phi\times\frac{b}{B}$$

$$= \frac{1}{B}\underset{\sim}{b}\times\nabla\phi\cdot\nabla n_e + n_e\,\nabla\cdot\left(\phi\,\nabla\times\left(\frac{b}{B}\right)\right)$$

$$\nabla\cdot\left[\frac{nT}{B}\,\frac{\underset{\sim}{B}\times\nabla B}{B^2}\right] = \nabla\cdot\left(P_e\,\nabla\times\left(\frac{b}{B}\right)\right)$$

$$\cancel{-\nabla\cdot\left(\frac{P_e}{B}\nabla\times\underset{\sim}{b}\right)}$$

define $$C(\phi) = \nabla\cdot\left(\phi\,\nabla\times\left(\frac{b}{B}\right)\right)$$

Density equation

$$\frac{\partial n}{\partial t} + \underbrace{\frac{1}{B}\underset{\sim}{b}\times\nabla\phi\cdot\nabla n}_{\text{advection}} + \underbrace{n\,C(\phi)}_{\text{compression}} + \underbrace{C(nT)}_{\text{Magnetic drift}} + \underbrace{\nabla\cdot\left(\underset{\sim}{b}V_{\parallel e}\, n\right)}_{\text{parallel flow}} = 0$$

$$\nabla\cdot\underset{\sim}{J} = 0$$

$$\underset{\sim}{J} = en\left(\underset{\sim}{V_i} - \underset{\sim}{V_e}\right)$$

$$\nabla\cdot\left[\frac{nm_i}{B^2}\,\frac{d E_\perp}{dt}\right] + \nabla\cdot\left(\underset{\sim}{b}J_\parallel\right) - C\big(e\underbrace{nT}_{P_e}\big) = 0$$

$$\underset{\sim}{E_\perp} \simeq -\nabla_\perp\phi \left[-\frac{\partial A_\perp}{\partial t}\right]$$

$$\nabla \cdot \left[\frac{n m_i}{B^2} \frac{d}{dt} \nabla_\perp \phi \right] = \nabla \cdot \left(\underset{\sim}{b} J_{\parallel} \right) - C(p_e)$$

Vorticity equation