CGL-MHD

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$\underline{\underline{P}}$ pressure tensor

MHD : $\underline{\underline{P}} = p\,\mathbb{I}$ $\begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix}$

Extended MHD $\underline{\underline{P}} = p\,\underline{\underline{\mathbb{I}}} + \underline{\underline{\Pi}}$ viscosity

2-fluid MHD $\underline{\underline{P}} = p_e \underline{\underline{\mathbb{I}}} + p_i \underline{\underline{\mathbb{I}}} + \underline{\underline{\Pi}}$

CGL $\underline{\underline{P}} = p_{\parallel} \underline{b}\underline{b} + p_{\perp}(\mathbb{I} - \underline{b}\underline{b}) + \underline{\underline{\Pi}}$

$\begin{pmatrix} p_{\perp} & & \\ & p_{\perp} & \\ & & p_{\parallel} \end{pmatrix} \leftarrow \underline{b}$

Timescales for pressure equilibriation $P_\parallel \sim P_\perp$

τ_e τ_i for electrons & ions

Core: $\tau_e \sim 10^{-4}s$ $\tau_i \sim 10^{-2}s$ $\Rightarrow$ anisotropic

Edge: $\tau_e \sim 10^{-6}s$ $\tau_i \sim 10^{-4}s$ $\Rightarrow$ anisotropic

(isotropic)

Closure

Ideal MHD adiabatic

$$\frac{d}{dt}\left(\frac{P}{\rho^\gamma}\right) = 0$$

CGL double adiabatic

$$\frac{d}{dt}\left(\frac{P_\parallel B^2}{\rho^3}\right) = 0$$

Assumes no heat flux

$$\frac{d}{dt}\left(\frac{P_\perp}{\rho B}\right) = 0 \qquad q_\parallel = q_\perp = 0$$

Conservation of adiabatic invariants

(1) conservation of $\mu = \frac{m v_\perp^2}{2B}$

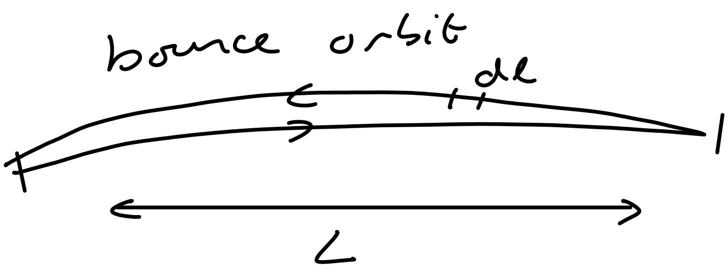
$$P_\perp \sim \langle V_\perp^2 \rangle \rho$$

$$V_\perp^2 \sim \mu B$$

$$\Rightarrow \quad P_\perp \sim \mu B \rho$$

$$\boxed{\frac{P_\perp}{\rho B} \sim \text{const}}$$

② Second adiabatic invariant

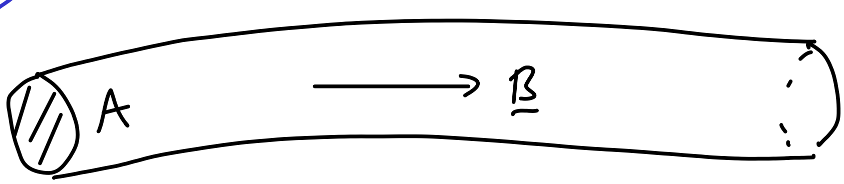


$$J = \oint m v_\parallel \, dl$$

$$J \sim m V_\parallel L$$

$$V_\parallel L \sim \text{const}$$

$$P_\parallel \sim \langle V_\parallel^2 \rangle \rho$$



flux $= AB \sim \text{const}$

Mass of trapped particles $\sim \rho L A \sim \text{const}$

$$L \sim \frac{1}{\rho A} \sim B/\rho$$

$$V_\parallel B/\rho \sim \text{const}$$

$$P_\parallel \sim \frac{\rho^2}{B^2} \rho \times \text{const}$$

$$\frac{P_\parallel B^2}{\rho^3} \sim \text{const}$$

CGL MHD

$$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s) = 0$$

s = species

$$P_\parallel \mathbf{b}\mathbf{b} + P_\perp (\mathbb{I} - \mathbf{b}\mathbf{b}) + \overset{\text{FLR}}{\underline{\underline{\Pi}}}$$

$$\frac{\partial \mathbf{u}_s}{\partial t} + \mathbf{u}_s \cdot \nabla \mathbf{u}_s + \frac{1}{\rho_s} \nabla \cdot \underline{\underline{P}}_s - \frac{q_s}{m_s}(\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) = 0$$

$$\frac{\partial P_{\parallel s}}{\partial t} + \nabla \cdot (\mathbf{u}_s P_{\parallel s}) + 2 P_{\parallel s} \mathbf{b} \cdot \nabla \mathbf{u}_s \cdot \mathbf{b} = 0$$

$$\frac{\partial P_{\perp s}}{\partial t} + \nabla \cdot (\mathbf{u}_s P_{\perp s}) + P_{\perp s} \nabla \cdot \mathbf{u}_s - P_\perp \mathbf{b} \cdot \nabla \mathbf{u}_s \cdot \mathbf{b} = 0$$