Firehose and Mirror instabilites

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Cold electron CGL MHD linear dispersion relation

$$\omega^4 - \omega^2 \underbrace{\left[\left(\frac{2P_\perp}{\rho_0} + V_A^2 \right) k_\perp^2 + \left(\frac{2P_\parallel}{\rho_0} + \frac{P_\perp}{\rho_0} + V_A^2 \right) k_\parallel^2 \right]}_{A_2}$$

$$+ 3k_\parallel^2 \frac{P_\parallel}{\rho_0} \underbrace{\left[\left(\frac{2P_\perp}{\rho_0} - \frac{P_\perp^2}{3P_\parallel \rho_0} + V_A^2 \right) k_\perp^2 - \left(\frac{P_\parallel}{\rho_0} - \frac{P_\perp}{\rho_0} - V_A^2 \right) k_\parallel^2 \right]}_{A_0} = 0$$

$V_A^2 = \frac{B^2}{\mu_0 \rho_0}$ $\quad \rho_0$ density $\quad P_\perp, P_\parallel$ pressures

$k_\parallel = \underset{\sim}{b} \cdot \underset{\sim}{k} \qquad \underset{\sim}{k_\perp} = \underset{\sim}{k} - k_\parallel \underset{\sim}{b}$

Parallel $k_\perp \ll k_\parallel$

$$\omega^2 = \frac{1}{\rho_0}\left(\frac{B^2}{\mu_0} - P_\parallel + P_\perp \right) k_\parallel^2 \qquad \text{Alfvén wave}$$

if $P_\perp = P_\parallel \Rightarrow$ MHD shear Alfvén wave if $P_\parallel > P_\perp + \frac{B^2}{\mu_0} \Rightarrow \omega^2 < 0$

instability

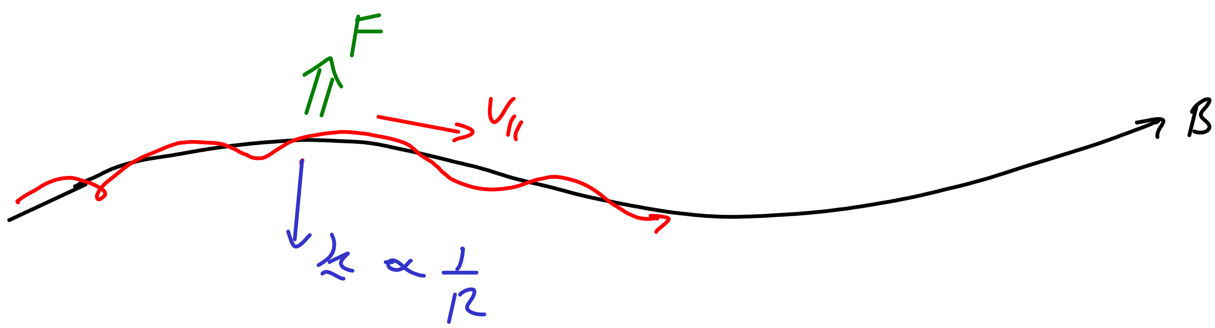
$\beta_\parallel = P_\parallel / (B^2/2\mu_0) \quad \beta_\perp = P_\perp / (B^2/2\mu_0)$

$\underline{\underline{\beta_\parallel > \beta_\perp + 2}}$ $\quad\quad$ $\underline{\underline{\beta_\parallel > 2}}$ (if $\beta_\perp = 0$)

$\Rightarrow$ space / astrophysics

not in magnetic confinement fusion

Mechanism: "centrifugal" instability



centrifugal force $\rightarrow$ particle drift

$\rightarrow$ can enhance perturbation (instability)

Stabilised by magnetic tension or perpendicular pressure

Mirror instability

Perpendicular "slow" wave $k_\perp \gg k_\parallel$

$$\omega^4 - A_2\omega^2 + A_0 = 0$$

$$\omega^2 = \frac{1}{2}\left(A_2 \pm \underbrace{\sqrt{A_2^2 - 4A_0}}_{>0}\right) \qquad A_2 > 0$$

if $\sqrt{A_2^2 - 4A_0} > A_2 \implies$ instability

$\implies A_0 < 0$ unstable

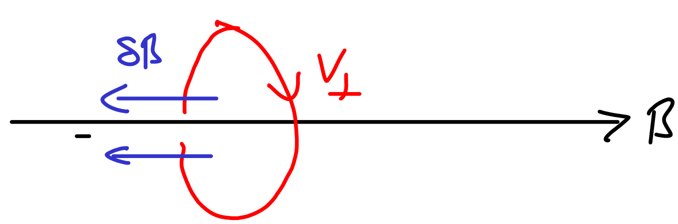
$$k_\parallel^2\left(1 - \tfrac{1}{2}\beta_\parallel + \tfrac{1}{2}a\beta_\parallel\right) + k_\perp^2\left(1 + a\beta_\parallel - \tfrac{1}{6}a^2\beta_\parallel\right) < 0$$

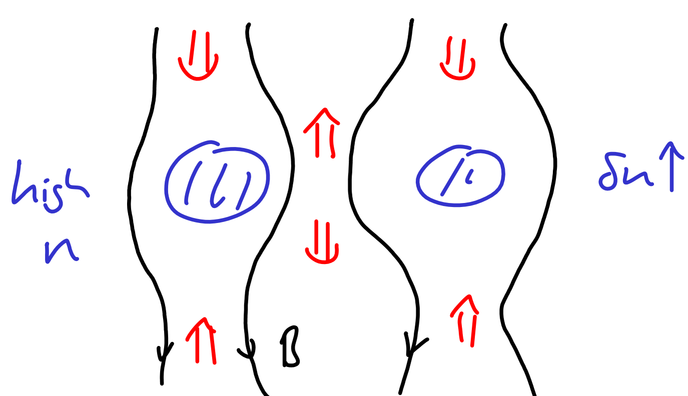
$a = \dfrac{T_\perp}{T_\parallel}$ anisotropy

$$\boxed{\frac{\beta_\perp^2}{\beta_\parallel} > 6(1+\beta_\perp)}$$ instability

$= 1$ in kinetic theory

Mechanism: "diamagnetic" instability

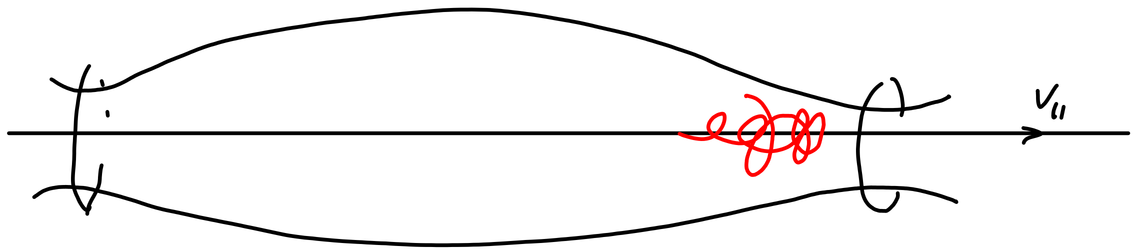




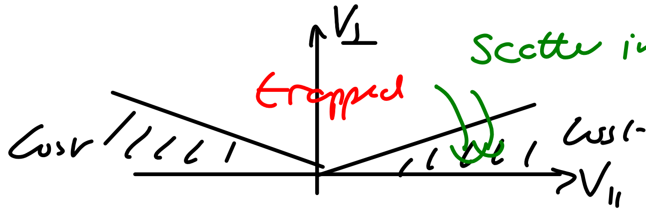
Perpendicular pressure reduces B field

$\implies$ creates a potential well and more trapping

δB, δn anti-correlated



Scatter into loss cone



$\Rightarrow$ Loss cone produces $P_\perp > P_\parallel$