# Equilibrium solutions

Concepts

- Force balance
- Hydrostatic equilibrium
- Viscous drag

Steady state $\frac{\partial}{\partial t} = 0$

$$\cancel{\frac{\partial \rho}{\partial t}} + \underset{\sim}{u} \cdot \nabla \rho = -\rho \nabla \cdot \underset{\sim}{u}$$

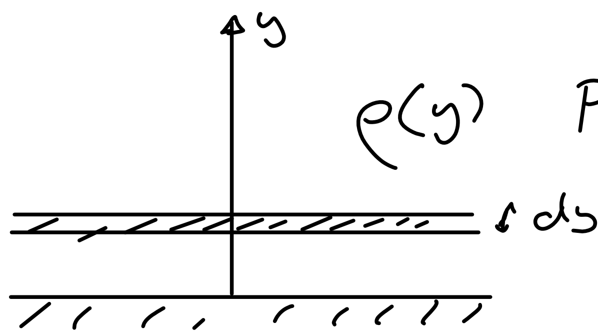
$$\rho \left[ \cancel{\frac{\partial \underset{\sim}{u}}{\partial t}} + \underset{\sim}{u} \cdot \nabla \underset{\sim}{u} \right] = \underline{-\nabla P + \underset{\sim}{F}}$$

$$\cancel{\frac{\partial P}{\partial t}} + \underset{\sim}{u} \cdot \nabla P = -\gamma P \nabla \cdot \underset{\sim}{u}$$

1) Hydro<u>static</u> $\underset{\sim}{u} = 0$

$$\underset{\sim}{F} = \rho \underset{\sim}{g}$$

$$\underset{\sim}{g} = -g\hat{y}$$




$\rho(y)$ $P(y)$

$$\underline{\frac{dP}{dy} = -g\rho}$$

$$PV = nk_BT$$

$$P = \left(\frac{n}{V}\right) k_BT \qquad \frac{n}{V} = \frac{\rho}{m_a}$$

$$p = \frac{\rho}{m_a} k_B T$$

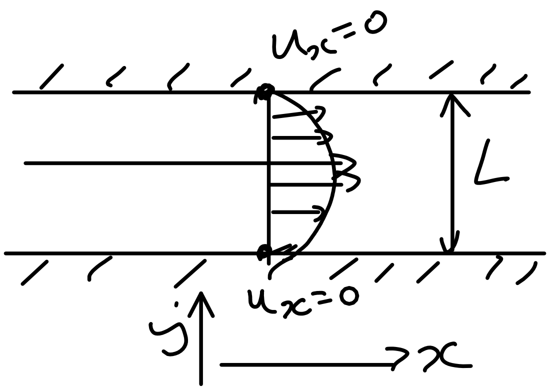
$$\frac{dp}{dy} = -g\frac{m_a}{k_B T}\, p$$

$$\underline{p = p_0\, e^{-\left(\frac{m_a g}{k_B T}\right) y}}$$

$$L = \frac{k_B T}{m_a g}$$

$\sim 8.5$ km
Earth atmosphere

## Viscous flow




$$\underset{\sim}{u} = u_x(y)\,\hat{x}$$

$$\rho\left[\frac{\partial \underset{\sim}{u}}{\partial t} + \underbrace{\underset{\sim}{u}\cdot\nabla \underset{\sim}{u}}_{=0}\right] = \underbrace{-\nabla p + \mu\nabla^2 \underset{\sim}{u}}_{\text{balance}} + \frac{1}{3}\mu\nabla(\cancel{\nabla\cdot \underset{\sim}{u}}) \to 0$$

$$\frac{dp}{dx} = \mu\frac{d^2}{dy^2}u_x$$

$$\frac{d^2 u_x}{dy^2} = \frac{1}{\mu}\frac{dp}{dx} \qquad u_x \propto y^2$$

Boundaries $u_x = 0$ at $y = 0$, $y = L$ no slip

$$u_x = -y(L-y)\frac{dP}{dx}\frac{1}{\mu}\frac{1}{2}$$

$$\frac{dP}{dx} < 0 \qquad u_x > 0$$

$$\cancel{\frac{\partial P}{\partial t}} + \underset{\sim}{u}\cdot\nabla P = \cancel{-\gamma P \nabla\cdot\underset{\sim}{u}} + \underbrace{\mu\, \underset{\sim}{u}\cdot\nabla^2\underset{\sim}{u} + \cancel{\frac{\mu}{3}\underset{\sim}{u}\cdot\nabla(\nabla\cdot\underset{\sim}{u})}}_{\text{work done}}$$

$0$

$$\underset{\sim}{u}\cdot\nabla P = u_x\frac{dP}{dx} < 0 \qquad \nabla\cdot u > 0\,?$$

$$= u_x\,\mu\,\frac{d^2u_x}{dy^2}$$