Irrotational flow

Concepts

- Static and dynamic pressure
- Irrotational flows
- Vorticity

$$\frac{\partial \underset{\sim}{u}}{\partial t} + \underset{\sim}{u} \cdot \nabla \underset{\sim}{u} = -\frac{1}{\rho} \nabla p$$

$$\underset{\sim}{u} \cdot \nabla \underset{\sim}{u} = (\nabla \times \underset{\sim}{u}) \times \underset{\sim}{u} + \nabla\left(\frac{1}{2} u^2\right)$$

Assume <u>incompressible</u> $\quad \nabla \cdot \underset{\sim}{u} = 0$

Density variation small

$$\frac{\partial \underset{\sim}{u}}{\partial t} + (\nabla \times \underset{\sim}{u}) \times \underset{\sim}{u} = -\nabla \underbrace{\left(\frac{p}{\rho} + \frac{1}{2} u^2\right)}_{H}$$

$$H\rho = \underbrace{p}_{\text{static pressure}} + \underbrace{\frac{1}{2}\rho u^2}_{\text{dynamic pressure}} \quad \text{total pressure}$$

① In steady state $\frac{\partial}{\partial t} \to 0$

$$0 = \underset{\sim}{u} \cdot \nabla H$$

H is constant along flow

② If $\nabla \times \underset{\sim}{u} = 0$ $\quad \nabla H = 0$

H is constant everywhere

irrotational flow

write $\underset{\sim}{u} = \nabla \phi$ ← Flow potential

$$\frac{\partial}{\partial t}(\nabla \phi) = -\nabla\left(\frac{P}{\rho} + \frac{1}{2}u^2\right)$$

$$\frac{\partial \phi}{\partial t} = -\frac{P}{\rho} - \frac{1}{2}u^2 + \cancel{f(t)} \rightarrow 0$$

Bernoulli's equation for unsteady flow

③ when $\nabla \times \underset{\sim}{u} \neq 0$

Vorticity