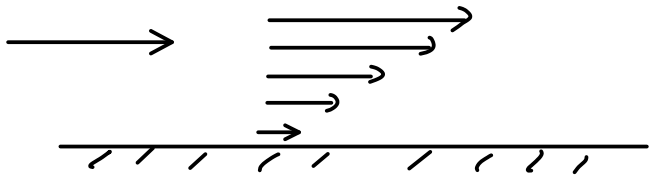
Vorticity

Concepts

- Local rotation and vorticity
- The vorticity equation
- 2D flow

Irrotational flow $\nabla \times \underset{\sim}{u} = 0$

$\underset{\sim}{\omega} = \nabla \times \underset{\sim}{u}$ vorticity



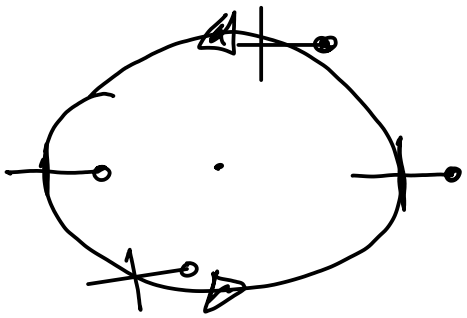
$\underset{\sim}{u} = (\beta y, 0, 0)$

$$\omega_x = \frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z} = 0$$

$$\omega_y = \frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x} = 0$$

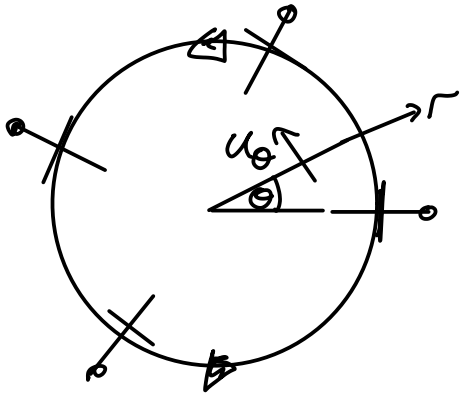
$$\omega_z = \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} = -\beta$$

no clear rotation, but have vorticity



no vorticity

$u_\theta \propto \frac{1}{r}$



with vorticity

$u_\theta \propto r$

Vorticity equation

$$\frac{\partial \underset{\sim}{u}}{\partial t} + \underset{\sim}{u} \cdot \nabla \underset{\sim}{u} = -\frac{1}{\rho} \nabla p$$

$$\frac{\partial \underset{\sim}{u}}{\partial t} + \underbrace{(\nabla \times \underset{\sim}{u})}_{\omega} \times \underset{\sim}{u} = -\nabla \left(\frac{p}{\rho} + \frac{1}{2} u^2 \right)$$

take curl note $\nabla \times \nabla( ) \to 0$

$$\frac{\partial \underset{\sim}{\omega}}{\partial t} + \nabla \times (\underset{\sim}{\omega} \times \underset{\sim}{u}) = 0$$

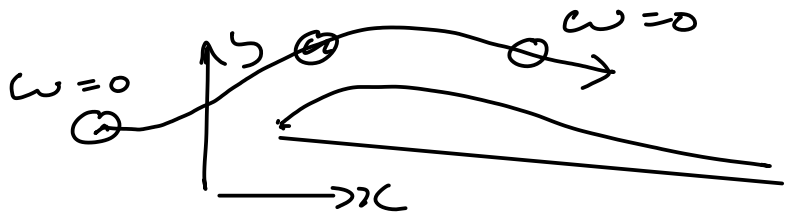
incompressible

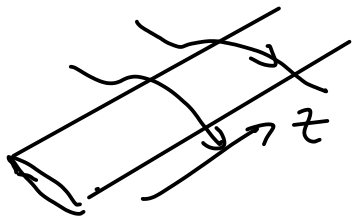
$\nabla \cdot \nabla \times ( ) = 0$

$$\frac{\partial \underset{\sim}{\omega}}{\partial t} + (\underset{\sim}{u} \cdot \nabla) \underset{\sim}{\omega} - (\underset{\sim}{\omega} \cdot \nabla) \underset{\sim}{u} + \underset{\sim}{\omega} \cancel{\nabla \cdot \underset{\sim}{u}} - \underset{\sim}{u} \cancel{(\nabla \cdot \underset{\sim}{\omega})}$$

$= 0$

$$\frac{\partial \underset{\sim}{\omega}}{\partial t} + \underset{\sim}{u} \cdot \nabla \underset{\sim}{\omega} = (\underset{\sim}{\omega} \cdot \nabla) \underset{\sim}{u}$$





2D Systems $\frac{\partial}{\partial z} = 0 \quad u_z = 0$

$$\omega_x = \omega_y = 0$$

$$\underset{\sim}{\omega} = \omega_z \hat{\underset{\sim}{k}}$$

$$\underset{\sim}{\omega} \cdot \nabla \underset{\sim}{u}$$

$$= \omega_z \frac{\partial}{\partial z} \underset{\sim}{u} \to 0$$

$$\frac{\partial \underset{\sim}{\omega}}{\partial t} + \underset{\sim}{u} \cdot \nabla \underset{\sim}{\omega} = 0$$

In 2D ideal incompressible flow
ω is constant along flow