Numerical solutions

Concepts

- Exact and approximate solutions
- Numerical methods
- Finite difference method
- Implementation using Python

Solution

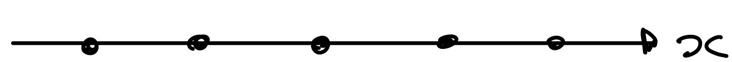
1. Exact analytic
2. Approximate analytic
3. Numerical solution

Numerical methods

Replace continuous equations & fields

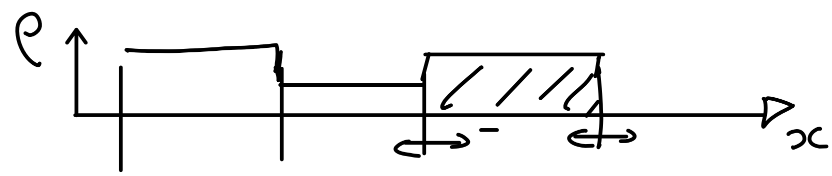
→ Discrete, finite set of values

Finite difference



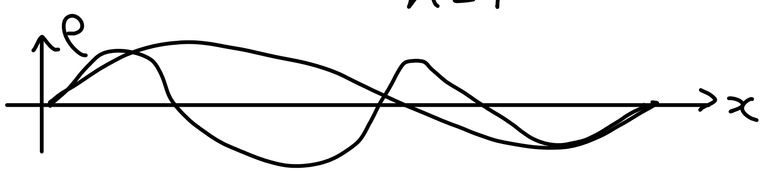
finite number of mesh points

Finite volume

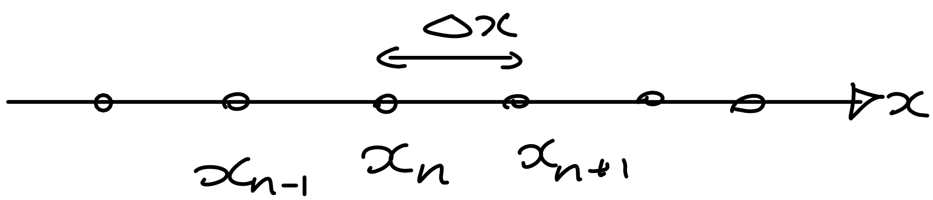


Spectral methods

$$\rho(t,x) = \sum_{n=1}^{N} \rho_n(t)\, \phi_n(x)$$



Finite difference



Taylor $\displaystyle \rho(x) \simeq \rho(x_n) + (x - x_n)\frac{\partial \rho}{\partial x}\Big|_{x_n} + \frac{1}{2!}(x-x_n)^2 \frac{\partial^2 \rho}{\partial x^2}\Big|_{x_n} + \cdots$

$x = x_{n+1}$

$$\underbrace{\rho(x_{n+1})}_{\text{known}} \simeq \underline{\rho(x_n)} + \Delta x \underbrace{\frac{\partial \rho}{\partial x}\Big|_{x_n}}_{\uparrow\ \text{find}} + \frac{\Delta x^2}{2!}\frac{\partial^2 \rho}{\partial x^2}\Big|_{x_n} + \cdots$$

$$\underbrace{\frac{\partial \rho}{\partial x}\Big|_{x_n} \simeq \frac{\rho(x_{n+1}) - \rho(x_n)}{\Delta x}}_{\text{Finite difference approximation}} \overbrace{- \frac{\Delta x}{2!}\frac{\partial^2 \rho}{\partial x^2}\Big|_{x_n} - \cdots}^{\text{Error}}$$

Density equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{u}) = 0$$

in 1D, $\underline{u}$ constant

$$\frac{\partial \rho}{\partial t} = -u_x \frac{\partial \rho}{\partial x} = -u_x \frac{\rho(x_{n+1}) - \rho(x_n)}{\Delta x}$$