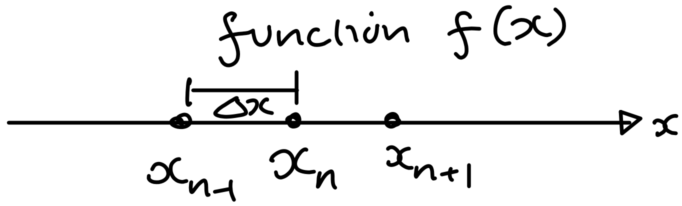
Central differencing

Contents

- Second order finite difference
- Solving Euler equations in 1D

Finite Difference



Taylor $$f(x) \simeq f(x_n) + (x - x_n)\left.\frac{\partial f}{\partial x}\right|_{x_n} + \frac{(x-x_n)^2}{2!}\left.\frac{\partial^2 f}{\partial x^2}\right|_{x_n} + \cdots$$

① $$f(x_{n-1}) \simeq f(x_n) - \Delta x\left.\frac{\partial f}{\partial x}\right|_{x_n} + \frac{\Delta x^2}{2!}\left.\frac{\partial^2 f}{\partial x^2}\right|_{x_n} - \frac{\Delta x^3}{3!}\left.\frac{\partial^3 f}{\partial x^3}\right|_{x_n} + \cdots$$

② $$f(x_{n+1}) \simeq f(x_n) + \Delta x\left.\frac{\partial f}{\partial x}\right|_{x_n} + \frac{\Delta x^2}{2!}\left.\frac{\partial^2 f}{\partial x^2}\right|_{x_n} + \frac{\Delta x^3}{3!}\left.\frac{\partial^3 f}{\partial x}\right|_{x_n} + \cdots$$

calculate

$$f(x_{n+1}) - f(x_{n-1}) \simeq 2\Delta x\left.\frac{\partial f}{\partial x}\right|_{x_n} + \frac{2\Delta x^3}{3!}\left.\frac{\partial^3 f}{\partial x^3}\right|_{x_n} + \cdots$$

$$\boxed{\left.\frac{\partial f}{\partial x}\right|_{x_n} \simeq \frac{f(x_{n+1}) - f(x_{n-1})}{2\Delta x}} \quad \underbrace{- \frac{\Delta x^2}{3!}\frac{\partial^3 f}{\partial x^3} - \cdots}_{\text{Error}}$$

central differencing

Error

Second order

$$f(x_{n+1}) + f(x_{n-1}) \simeq 2f(x_n) + 2\frac{\Delta x^2}{2!}\frac{\partial^2 f}{\partial x^2} + \frac{2\Delta x^4}{4!}\frac{\partial^4 f}{\partial x^4}$$

$$\boxed{\frac{\partial^2 f}{\partial x^2} \simeq \frac{f(x_{n+1}) - 2f(x_n) + f(x_{n-1})}{\Delta x^2}} + \underbrace{\frac{2\Delta x^2}{4!}\frac{\partial^4 f}{\partial x^4}}_{\text{Error}}$$

central difference

1D Euler

momentum

flow speed $\underset{\sim}{u}$

$$\frac{\partial \underset{\sim}{u}}{\partial t} + \underset{\sim}{u}\cdot\nabla\underset{\sim}{u} = -\frac{1}{\rho}\nabla P$$

pressure

mass density $\sim$ const

pressure P

$$\frac{\partial P}{\partial t} + \underset{\sim}{u}\cdot\nabla u = -\gamma P \nabla\cdot\underset{\sim}{u}$$

1D $\quad \underset{\sim}{u} = u_x \hat{x}$