Ideal MHD

Contents

- Review neutral fluid (Euler) equations
- Low frequency electromagnetic forces
- Assumptions
- Some applications

Euler equations

mass density ρ

$$\frac{\partial \rho}{\partial t} + \underset{\sim}{u} \cdot \nabla \rho = -\rho \nabla \cdot \underset{\sim}{u}$$

flow velocity $\underset{\sim}{u}$

$$\rho \left[\frac{\partial \underset{\sim}{u}}{\partial t} + \underset{\sim}{u} \cdot \nabla \underset{\sim}{u} \right] = -\nabla P + F \qquad [N/m^3]$$

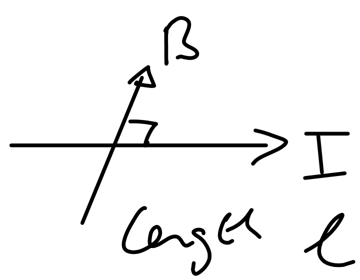
F: $\rho \underset{\sim}{g}$ gravity, viscosity

pressure P

$$\frac{\partial P}{\partial t} + \underset{\sim}{u} \cdot \nabla P = -\gamma P \nabla \cdot \underset{\sim}{u}$$

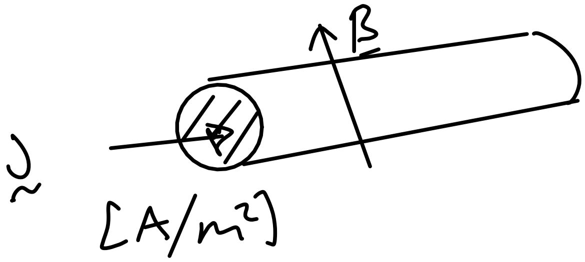
Force on conducting fluid

wire



force $= B I l$

$[N]$



$$\underline{F} = \underline{J} \times \underline{B}$$
$[N/m^3]$

$$\rho\left[\frac{\partial \underline{u}}{\partial t} + \underline{u}\cdot\nabla\underline{u}\right] = -\nabla p + \underline{\underline{\underline{J} \times \underline{B}}}$$

Calculating $\underline{J}$, B

Assume $\underline{E} = 0$ in frame of fluid ($\underline{u} = 0$)

i.e. no resistivity

$$\underline{E} = \underline{E}' + \underline{u} \times \underline{B} = 0 \quad \text{in lab frame}$$

IDEAL MHD OHM'S LAW

lab frame

$$\frac{\partial \underline{B}}{\partial t} = -\nabla \times \underline{E} \quad \text{Faraday's law}$$

$$= \nabla \times (\underline{u} \times \underline{B})$$

$$\boxed{\nabla \times \underline{B} = \mu_0 \underline{J}} + \underbrace{\cancel{\mu_0 \varepsilon_0 \frac{\partial \underline{E}}{\partial t}}}_{\text{Displacement current}} \quad \text{Ampère's law}$$

$\Rightarrow$ neglect (low frequency)

$$\boxed{\nabla \cdot \underset{\sim}{J} = 0}$$

Quasineutrality

no net charge

Summary

$\rho, \underline{u}, P, \underline{B}$ — closed set of 8 equations
(7 if $\nabla \cdot B = 0$ is included)

$$\frac{\partial \rho}{\partial t} + \underline{u} \cdot \nabla \rho = -\rho \nabla \cdot \underline{u}$$

$$\rho \left(\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u} \right) = -\nabla P + \underline{J} \times \underline{B}$$

$$\frac{\partial P}{\partial t} + \underline{u} \cdot \nabla P = -\gamma P \nabla \cdot \underline{u}$$

$$\underline{J} = \frac{1}{\mu_0} \nabla \times \underline{B}$$

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{u} \times \underline{B})$$

Main assumptions

- Frequent collision (LTE)
- Length scales $\gg r_L$ Larmor radius
- Frequencies $\ll$ cyclotron frequencies $\ll$ collision frequency
- Quasineutral plasma