Single fluid equations

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- Low frequency approximation
- Quasineutrality
- Collisional regime

$$\frac{\partial n_\alpha}{\partial t} + \nabla\cdot(\underset{\sim}{u}_\alpha n_\alpha) = 0$$

α species
e.g. e, i

$$\frac{\partial}{\partial t}(\underset{\sim}{u}_\alpha n_\alpha) + \nabla\cdot(n_\alpha \underset{\sim}{u}_\alpha \underset{\sim}{u}_\alpha) + \frac{1}{m_\alpha}\nabla\cdot\underline{\underline{P}}_\alpha = \frac{q_\alpha}{m_\alpha} n_\alpha(\underset{\sim}{E} + \underset{\sim}{u}_\alpha \times \underset{\sim}{B}) + \underset{\sim}{R}_\alpha$$

Maxwell

$\nabla\cdot\nabla\times B = 0 \Rightarrow \nabla\cdot J = 0$

$$\nabla\times E = -\frac{\partial B}{\partial t} \qquad \boxed{\nabla\times B = \mu_0 \underset{\sim}{J}} + \cancel{\mu_0\varepsilon_0\frac{\partial E}{\partial t}}$$

neglected
$V_{th} << c$
$\omega/k << c$

$\sum_\alpha q_\alpha n u_\alpha$

$$\nabla\cdot\underset{\sim}{E} = \frac{\bar{Q}}{\varepsilon_0} \qquad \nabla\cdot\underset{\sim}{B}$$

charge density $\sum_\alpha q_\alpha n_\alpha$

$\varepsilon_0 \to 0$

$$\varepsilon_0\nabla\cdot\underset{\sim}{E} = \bar{Q} \longrightarrow 0$$

Quasi-neutrality

$\omega << \omega_{pe}$
freq plasma freq.

$\nabla\cdot\underset{\sim}{E} \neq 0$

$\lambda >> \lambda_D$ Debye length

$$\frac{\bar{Q}}{q_e n_e} << 1$$

Mass density

$$m_i \gg m_e \qquad m_e \to 0$$

$$\rho = m_i n_i + \cancel{m_e n_e} + \cdots$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{u}_i) = 0$$

Momentum

Add equations for each species

$$\frac{\partial}{\partial t}(\underbrace{m_i n_i u_i}_{\rho} + \cancel{m_e n_e u_e} + \cdots)$$

$$\boxed{\frac{\partial}{\partial t}(\rho \underline{u}_i) + \nabla \cdot (\rho \underline{u}_i \underline{u}_i) + \nabla \cdot (\underbrace{\underline{\underline{P}}_e + \underline{\underline{P}}_i}_{P}) = \underline{J} \times \underline{B}}$$

adiabatic

$$\left(\frac{P}{\rho}\right)^{\gamma} = \text{const}$$

Collisional regime $\omega \ll \nu$

$$\lambda \gg \lambda_{mfp}$$

$\underline{\underline{P}} \to P$ isotropic pressure (scalar)

Single fluid $P_e = P_i$ $\quad T_e = T_i$

$$\frac{\lambda_{mfp}}{\lambda} \ll \sqrt{\frac{m_e}{m_i}}$$

Ohm's law

electron momentum

resistivity

$$m_e \to 0 \qquad \nabla \cdot \underline{\underline{P}}_e = -e n_e (\underline{E} + \underline{u}_e \times \underline{B}) + \underline{R}_e$$

$$\underline{\underline{P}}_e \to p$$

$$\underline{J} = e n_i \underline{u}_i - e n_e \underline{u}_e$$

$$\underline{u}_e = \underline{u}_i - \frac{\underline{J}}{en} \qquad n_i = n_e$$

quasi-neutrality

$$\nabla p_e + en(\underline{E} + \underline{u}_i \times \underline{B}) - \underline{J} \times \underline{B} + \underline{R}_e$$

$$\underline{E} + \underline{u}_i \times \underline{B} = \frac{1}{en}\left(\underbrace{\underline{J} \times \underline{B}}_{\text{Hall term}} - \underbrace{\nabla p_e}_{\text{electron diamagnetic}} + \underline{R}_e\right)$$

$\omega \ll \Omega_i$ ion cyclotron

$\lambda \gg r_{Li}$ gyro-radius

$$\boxed{\underline{E} + \underline{u}_i \times \underline{B} = 0}$$

Ideal MHD Ohm's Law