Equilibrium solutions

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- Force balance in ideal MHD
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- Magnetic pressure

$$\cancel{\frac{\partial \rho}{\partial t}} + \cancel{\underline{u}\cdot\nabla\rho} = -\cancel{\rho\nabla\cdot\underline{u}}$$

$$\rho\left[\cancel{\frac{\partial \underline{u}}{\partial t}} + \cancel{\underline{u}\cdot\nabla\underline{u}}\right] = \boxed{-\nabla P + \underset{\sim}{J}\times\underset{\sim}{B}}$$

$$\nabla\times\underline{B} = \mu_0\underline{J}$$

$$\cancel{\frac{\partial P}{\partial t}} + \cancel{\underline{u}\cdot\nabla P} = -\cancel{\gamma P\nabla\cdot\underline{u}}$$

$$\cancel{\frac{\partial \underline{B}}{\partial t}} = \cancel{\nabla\times(\underline{u}\times\underline{B})}$$

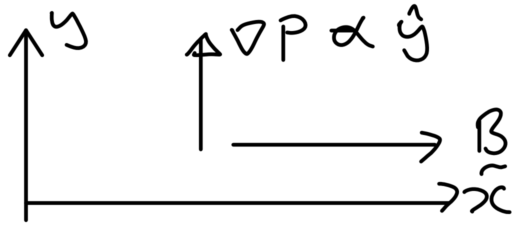
stationary equilibria

$\underset{\sim}{u} = 0$ $\qquad \frac{\partial}{\partial t}\to 0$

Force balance: $\boxed{\underline{J}\times\underline{B} = \nabla P}$

$\underset{\sim}{B}\cdot(\underset{\sim}{J}\times\underline{B}) = 0 = \underset{\sim}{B}\cdot\nabla P$ no pressure gradient along $\underline{B}$

$\underset{\sim}{J}\cdot\nabla P = 0$ no current in ∇P direction



$\underset{\sim}{B} = B_x \underset{\sim}{\hat{x}}$ $\otimes \underset{\sim}{J} = J_z \underset{\sim}{\hat{z}}$

$$\underset{\sim}{J} = \frac{1}{\mu_0} \nabla \times \underset{\sim}{B}$$

$$J_x = \frac{1}{\mu_0}\left(\cancel{\frac{\partial B_y}{\partial z}} - \cancel{\frac{\partial B_z}{\partial y}}\right) = 0$$

$$J_y = \frac{1}{\mu_0}\left(\cancel{\frac{\partial B_z}{\partial x}} - \cancel{\frac{\partial B_x}{\partial z}}\right) = 0$$

$$J_z = \frac{1}{\mu_0}\left(\underline{\underline{\frac{\partial B_x}{\partial y}}} - \cancel{\frac{\partial B_y}{\partial x}}\right) = \underline{\underline{\frac{1}{\mu_0}\frac{\partial B_x}{\partial y}}}$$

$$\underset{\sim}{J} \times \underset{\sim}{B} = \frac{1}{\mu_0} \underbrace{\frac{\partial B_x}{\partial y} B_x}_{\frac{\partial}{\partial y}\left(\frac{B_x^2}{2}\right)} \underbrace{\underset{\sim}{\hat{z}} \times \underset{\sim}{\hat{x}}}_{-\underset{\sim}{\hat{y}}} = -\frac{\partial}{\partial y}\left(\frac{B_x^2}{2\mu_0}\right)\underset{\sim}{\hat{y}}$$

$$= \frac{\partial P}{\partial y}\underset{\sim}{\hat{y}}$$

$$\boxed{\frac{\partial}{\partial y}\left(P + \frac{B_x^2}{2\mu_0}\right) = 0}$$

fluid pressure

Magnetic pressure